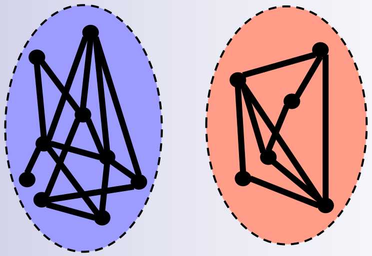

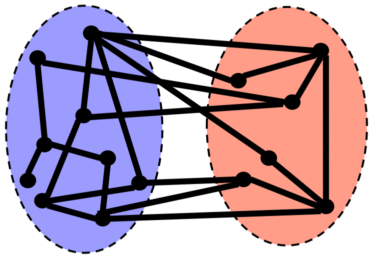

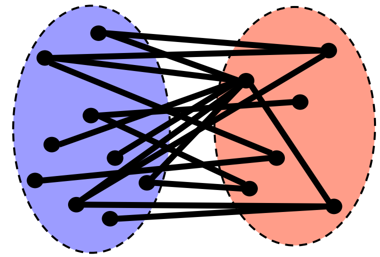

To mix or not to mix?

- Desirable and undesirable levels of segregation between groups

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https://luis-r-izquierdo.github.io/micopro/

Outline

- The question
- Related literature
- The approach and the model
- A mean-dynamics (MD) approximation
- Analysis of the MD
- Robustness
- Conclusions

The question

How does mixing affect contagion processes?

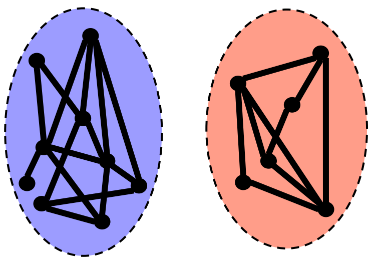
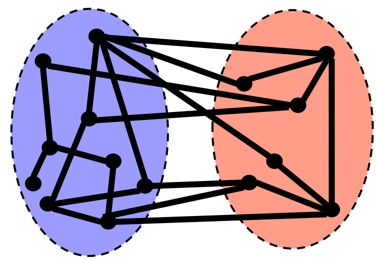
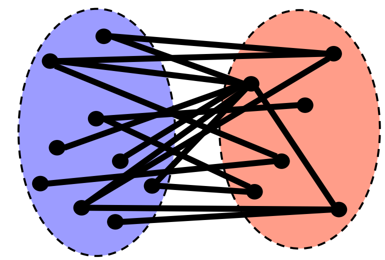
homophily

assortativity

segregation

diffusion

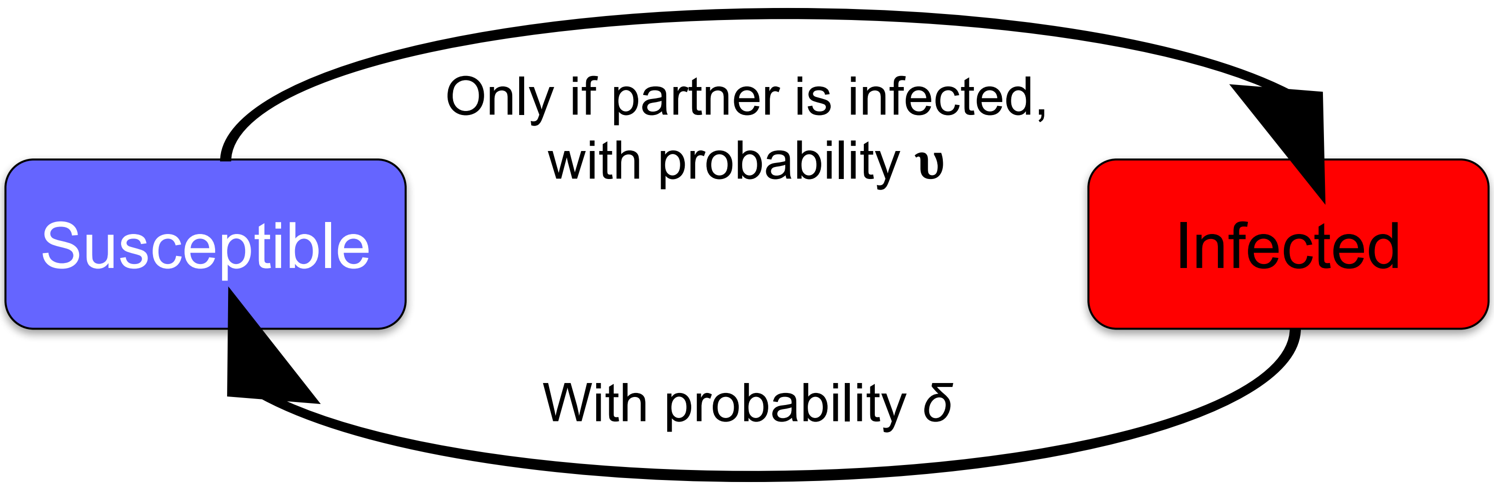
The question

Homophily	Random	Heterophily
Assortativity	Random	Disassortativity
Segregation	Random	No segregation
100% intragroup	50% intragroup	0% intragroup
0% intergroup	50% intergroup	100% intergroup
No mixing	**Some mixing**	**Full mixing**

The question

How does mixing affect contagion processes?
diffusion

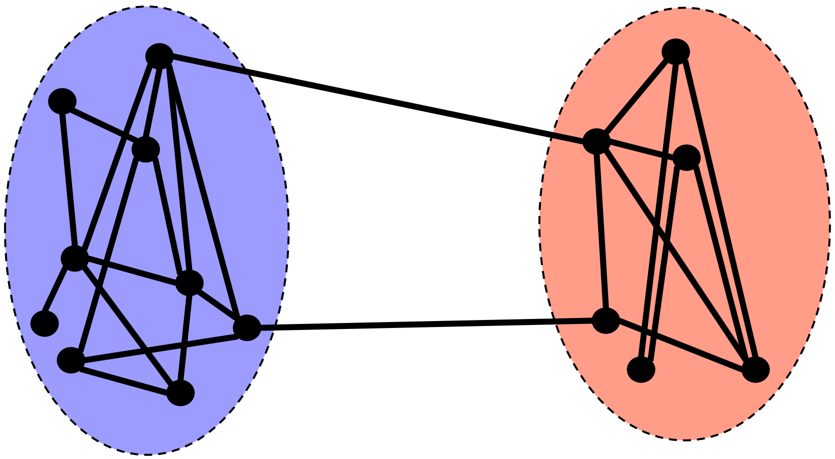


Some examples:

- Biological infections (e.g. common cold and influenza)
- Adoption of the latest technology
- Motivation in the classroom or at work

The question

How does mixing affect contagion processes?



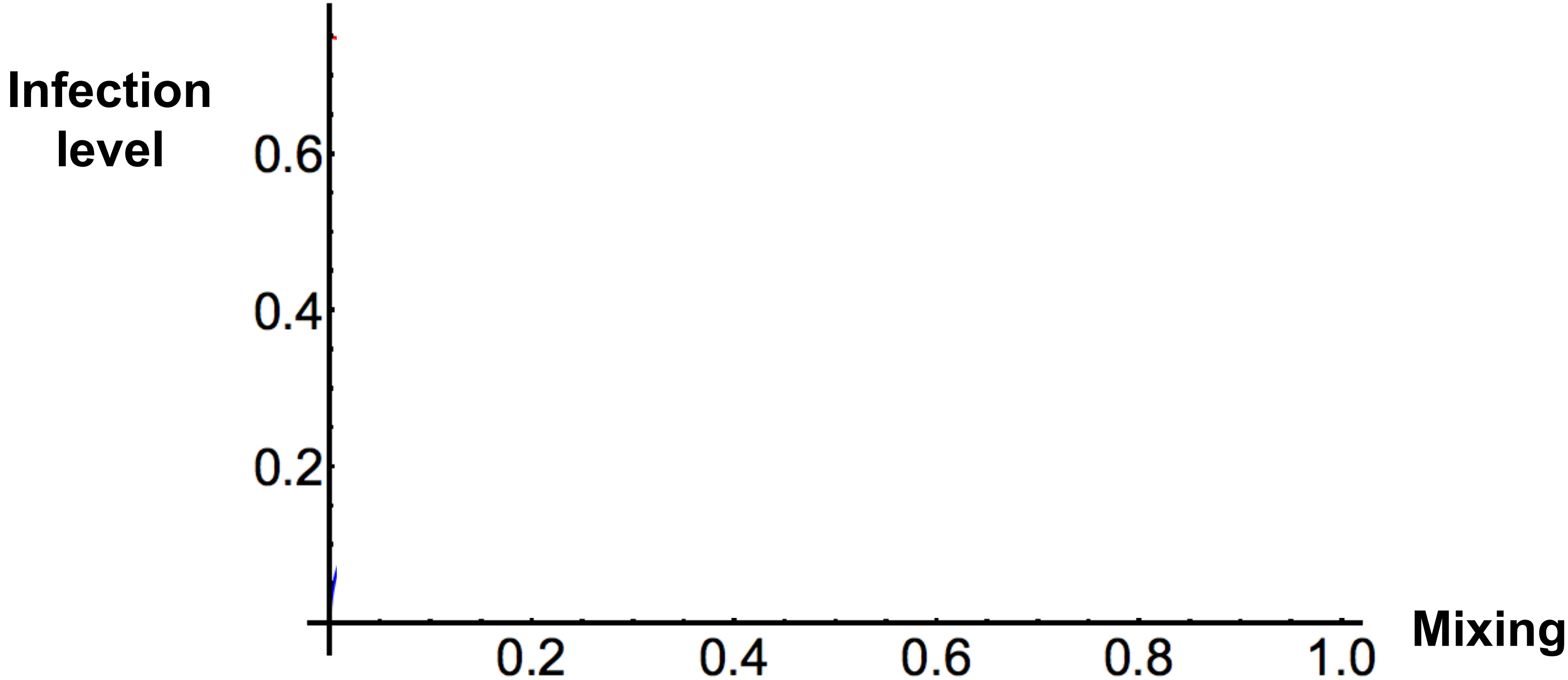
Resistant group

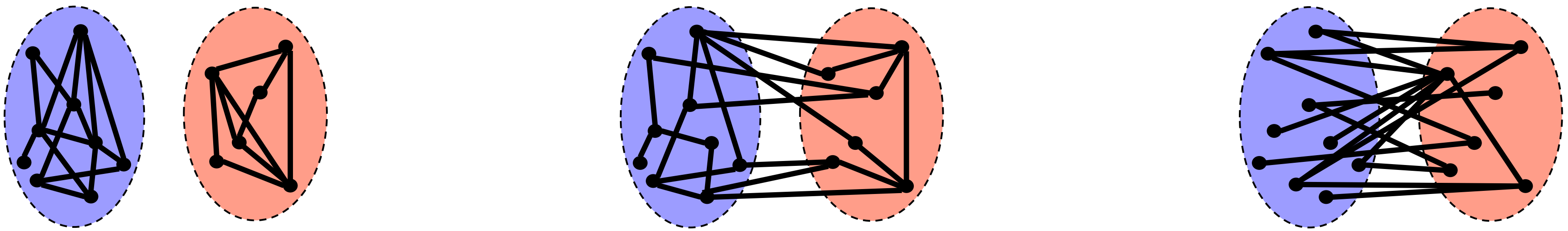
Hard to infect / motivate

Sensitive group

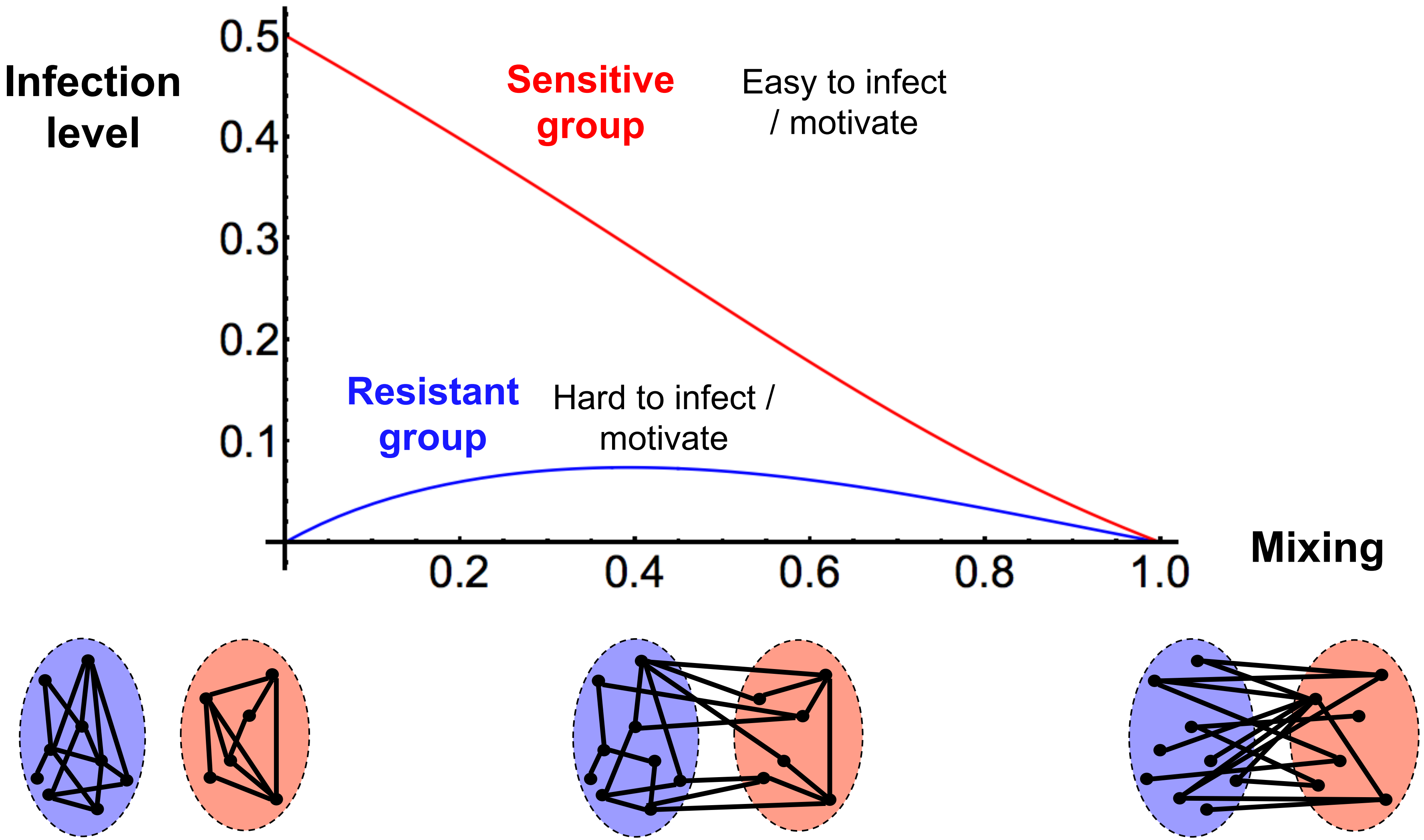
Easy to infect / motivate

The question





The question



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Related literature

- Axelrod, R. (1997) The dissemination of culture - A model with local convergence and global polarization. *Journal of Conflict Resolution* 41(2), 203-226.
- McPherson, M., Smith-Lovin, L., & Cook, J. M. (2001). Birds of a feather: Homophily in social networks. *Annual Review of Sociology*, 27, 415–444.
- Jackson, M. O. & López-Pintado, D. (2013). Diffusion and contagion in networks with heterogeneous agents and homophily. *Network Science*, 1, 49-67.
- Yavaş, M., & Yücel, G. (2014). Impact of Homophily on Diffusion Dynamics over Social Networks. *Social Science Computer Review*, 32 (3), 354–372.

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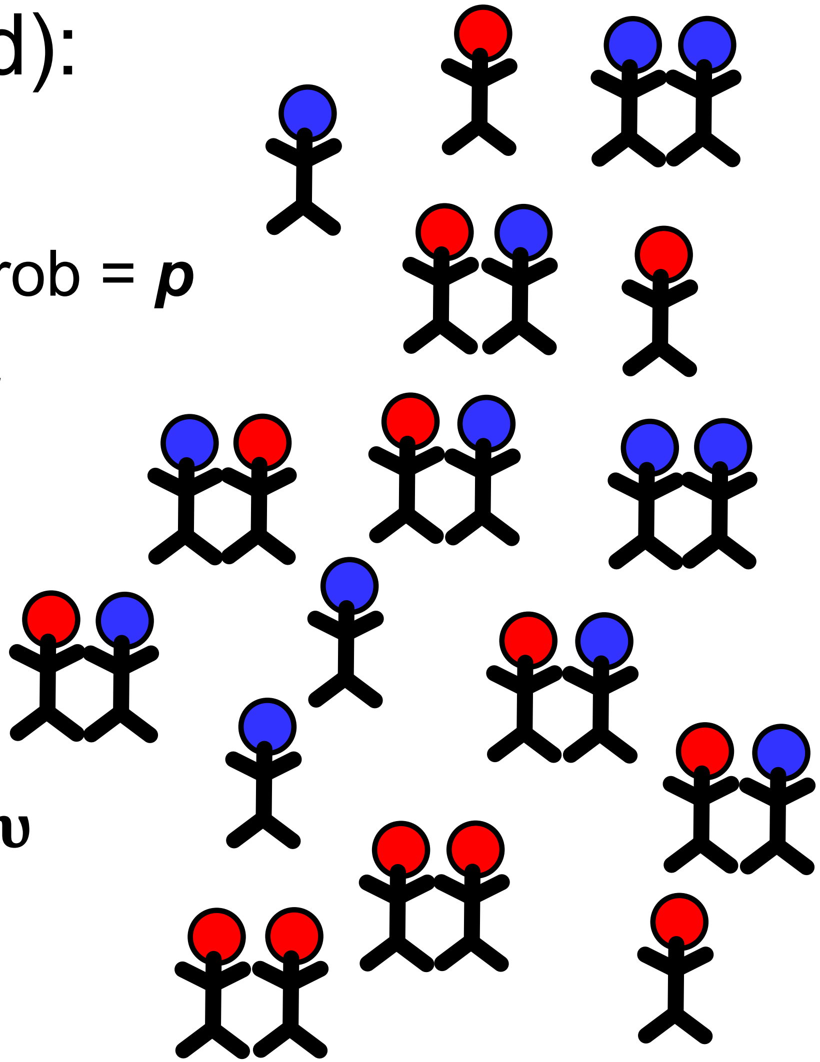
The model

Matching model (undirected):

1. Two groups of equal size
2. Agents selected to interact with prob = ***p***
3. Do matching according to ***mixing***
4. For each agent:
   - If infected:
     recover with prob = **δ**
   - Else: if mate infected,
     become infected with prob = **υ**



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A mean-dynamics (MD) approximation

When the population is very large, the law of large numbers enables us to:

- identify *expected* recoveries with *average* recoveries, and
- identify *expected* infections with *average* infections.

$$\frac{d\rho_1}{dt} = p(1-\rho_1)v_1\big((1-m)\rho_1 + m\rho_2\big) - \rho_1\delta_1$$

$$\frac{d\rho_2}{dt} = p(1-\rho_2)v_2\big((1-m)\rho_2 + m\rho_1\big) - \rho_2\delta_2$$

A mean-dynamics (MD) approximation

When the population is very large, the law of large numbers enables us to:

- identify *expected* recoveries with *average* recoveries, and
- identify *expected* infections with *average* infections.

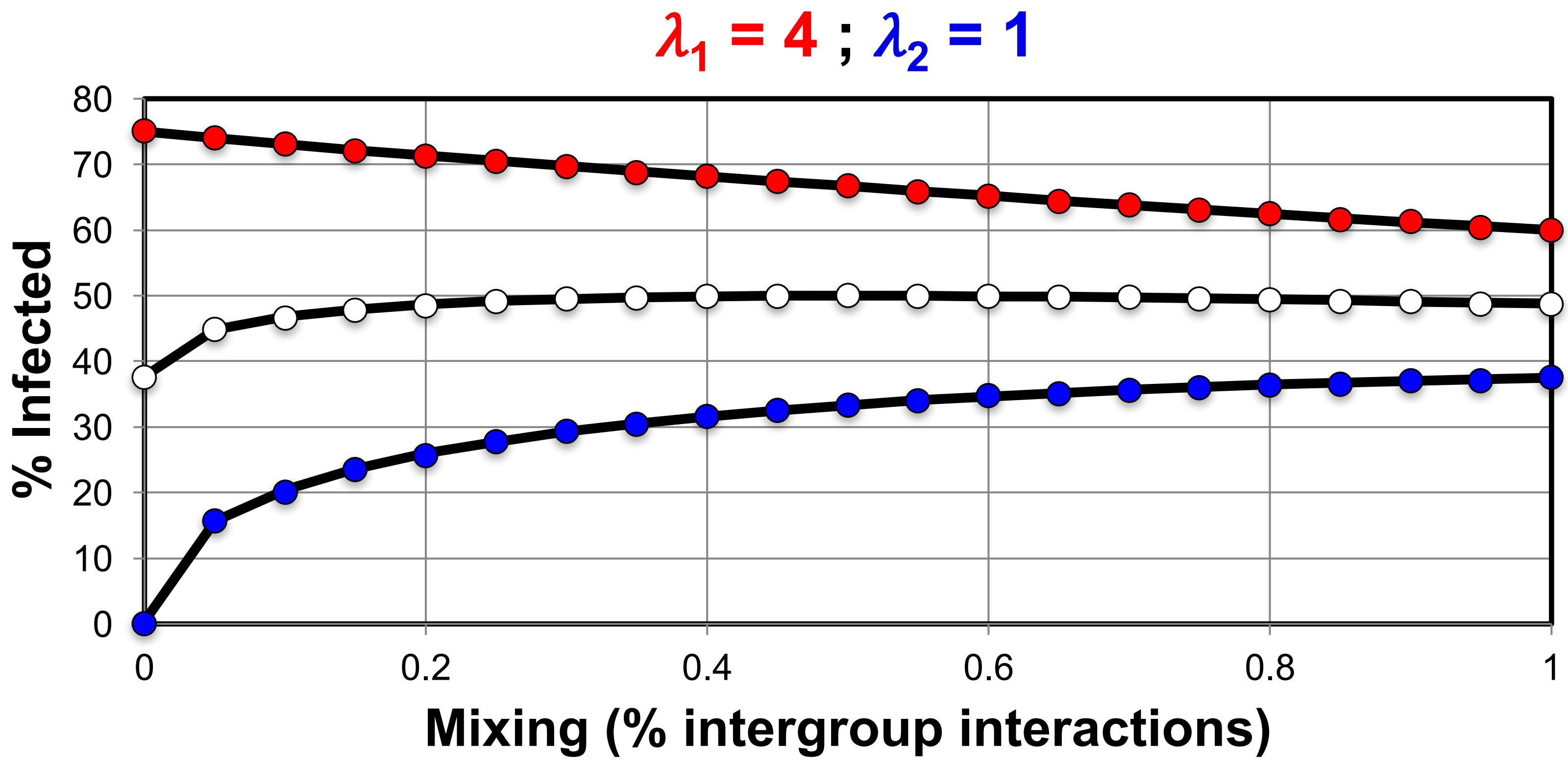
$$\frac{1}{\delta_1}\frac{d\rho_1}{dt} = \lambda_1(1-\rho_1)\big((1-m)\rho_1 + m\rho_2\big) - \rho_1$$

$$\frac{1}{\delta_2}\frac{d\rho_2}{dt} = \lambda_2(1-\rho_2)\big((1-m)\rho_2 + m\rho_1\big) - \rho_2$$

$$\lambda_i = p\frac{v_i}{\delta_i}$$

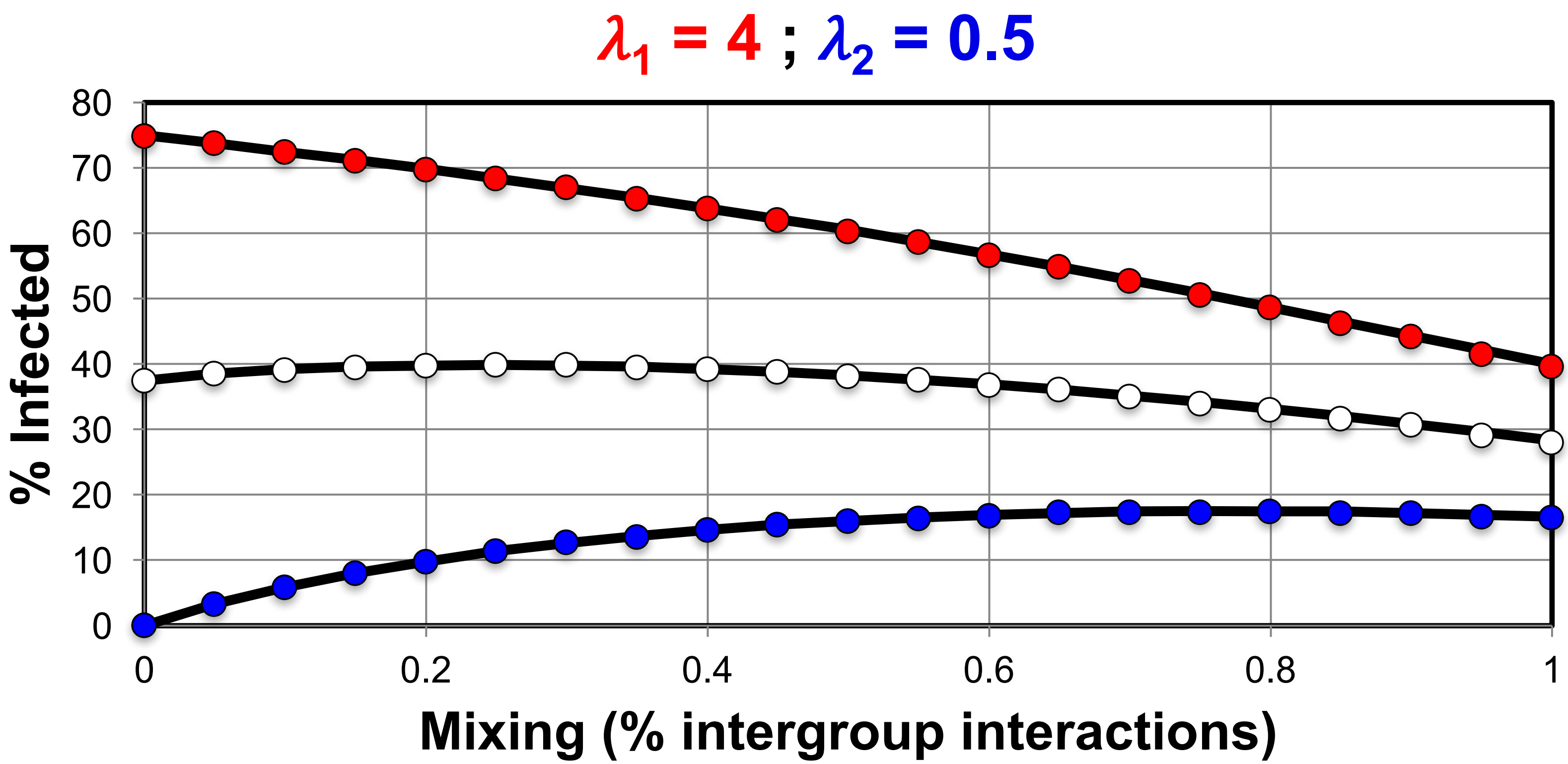
Effective adoption rate

A mean-dynamics (MD) approximation



Circles are calculated using the ABM (1000 agents). Each circle is the average of 10 runs. For each run we have computed the average of the % infected from t = 1001 to t = 2000. Standard errors are below 1% in all cases.
Black lines are computed using the MD.

A mean-dynamics (MD) approximation



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Analysis of the MD

$$\frac{1}{\delta_1}\frac{d\rho_1}{dt} = \lambda_1(1-\rho_1)\big((1-m)\rho_1 + m\rho_2\big) - \rho_1 = 0$$

$$\frac{1}{\delta_2}\frac{d\rho_2}{dt} = \lambda_2(1-\rho_2)\big((1-m)\rho_2 + m\rho_1\big) - \rho_2 = 0$$

- Existence and uniqueness of a stable solution, for all parameter values, regardless of initial conditions.

Analysis of the MD

Result 1. Assume $\lambda_1 < \lambda_2$. A unique solution $(\rho_1^s, \rho_2^s) \neq (0,0)$ exists iff

$$\lambda_2(1-m) \geq 1 \qquad \text{[a]}$$

or $$\frac{\lambda_1 m}{1-\lambda_1\,(1-m)} \frac{\lambda_2 m}{1-\lambda_2\,(1-m)} > 1 \qquad \text{[b]}$$

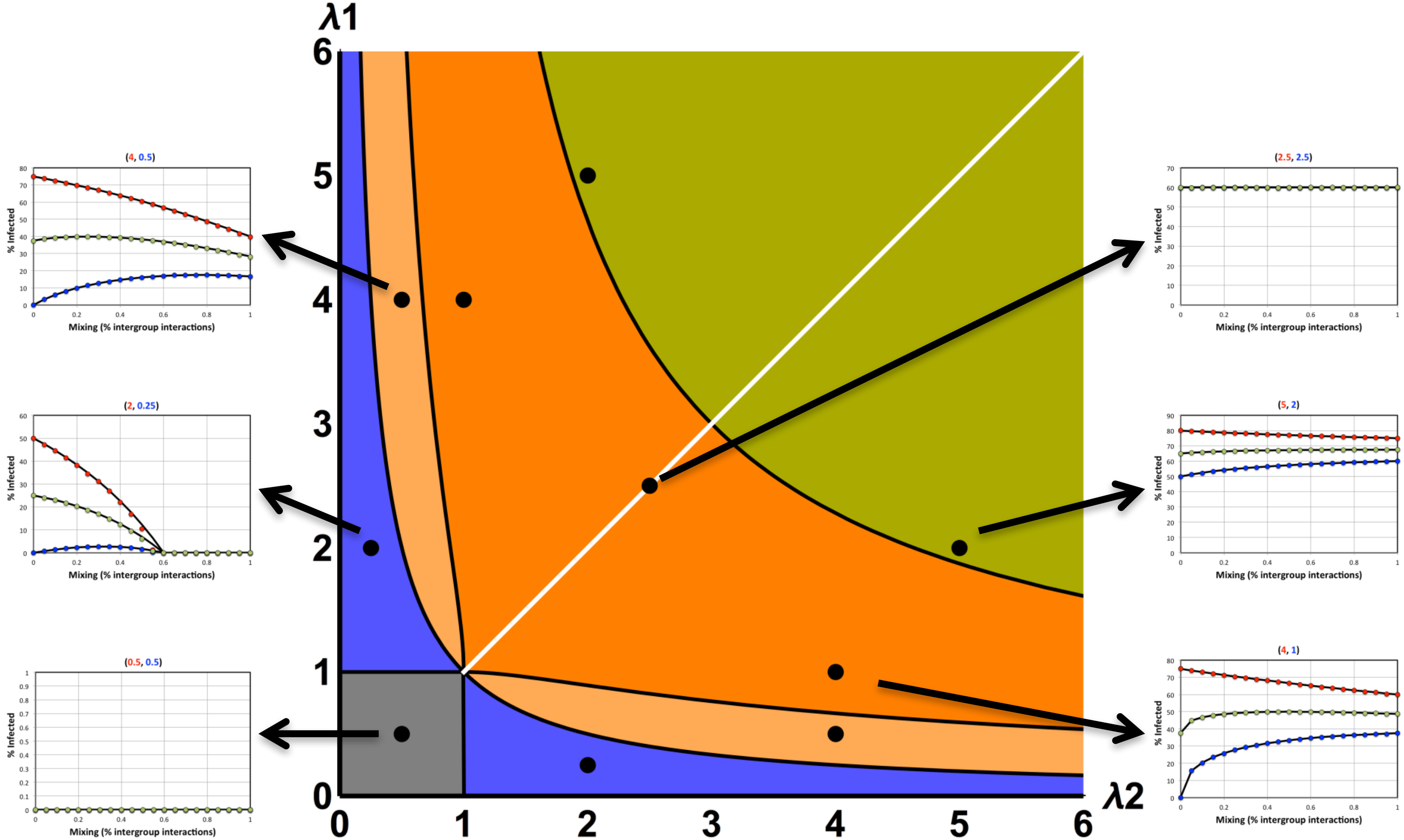
Corollary 1.1: If $\lambda_1\lambda_2 > 1$ then there is a unique positive solution.

Corollary 1.2: If $\lambda_2 \leq 1$ then (ρ_1, ρ_2) = (0, 0) is the only solution.

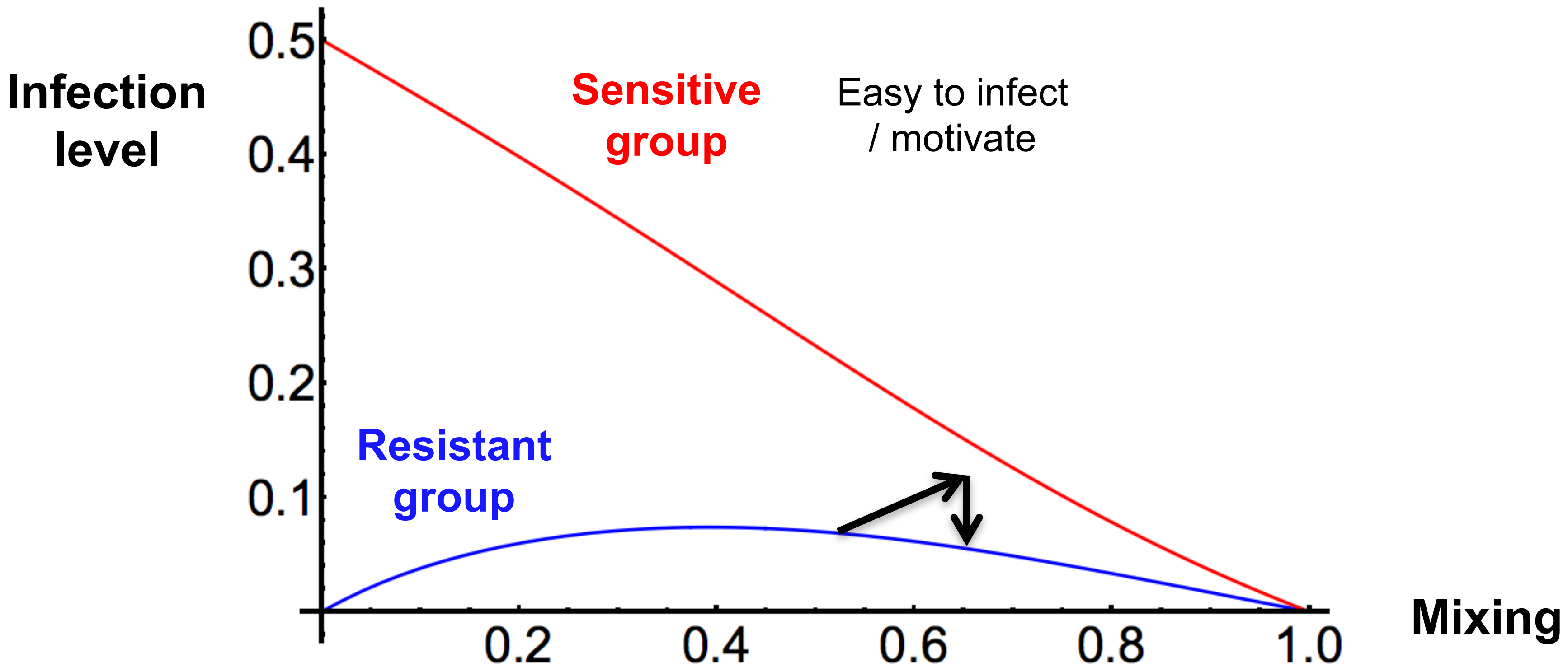
Corollary 1.3: If $\lambda_2 > 1$ but $\lambda_1\lambda_2 \leq 1$ then there is a unique positive solution iff the mixing level m is lower than some critical level

$$m_{critical} = \frac{\lambda_1 + \lambda_2 - \lambda_1\lambda_2 - 1}{\lambda_1 + \lambda_2 - 2\lambda_1\lambda_2}$$

Analysis of the MD



Analysis of the MD



$$d\rho_1 = \frac{\partial \rho_1}{\partial m} dm + \frac{\partial \rho_1}{\partial \rho_2} d\rho_2$$

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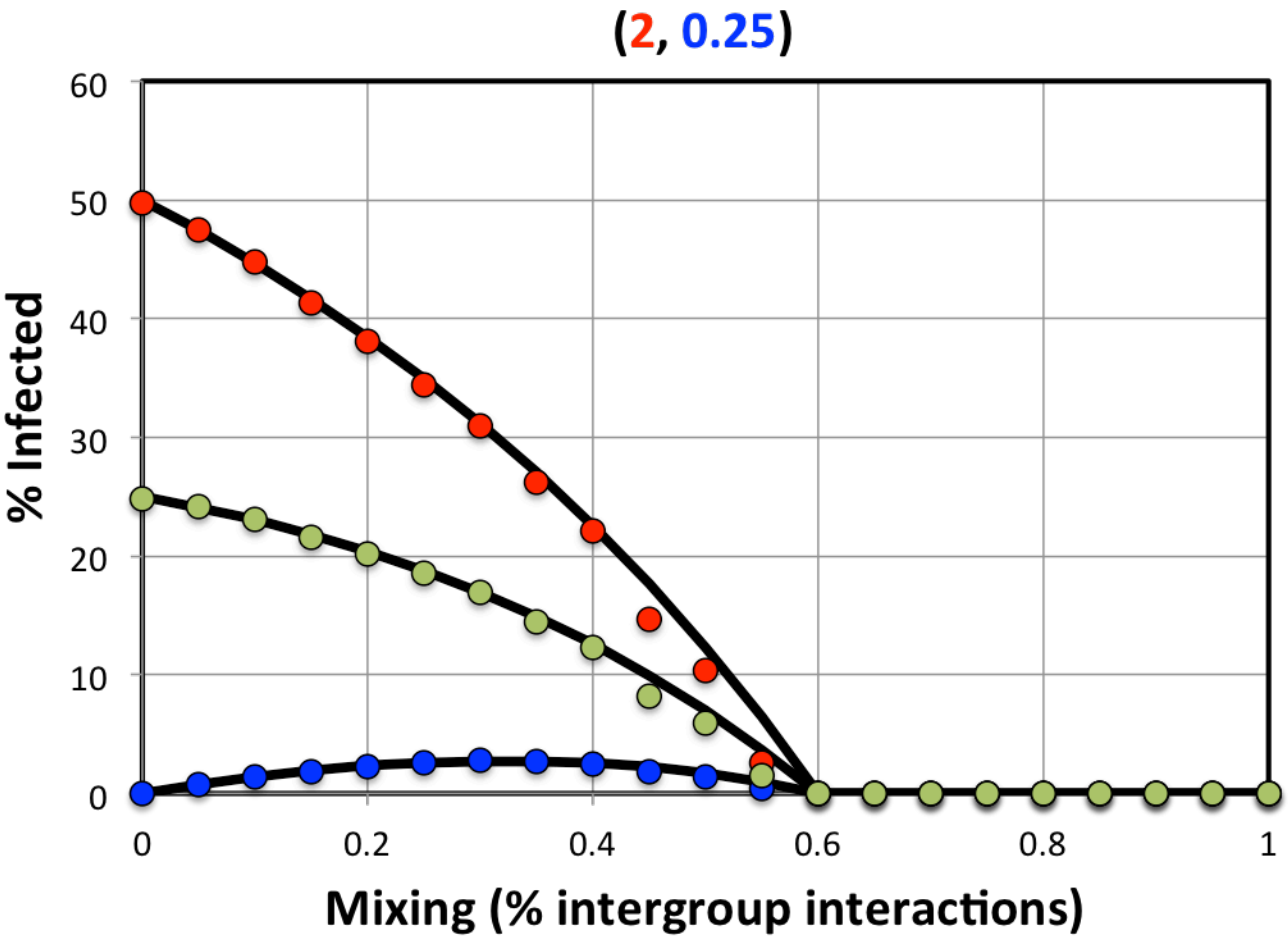
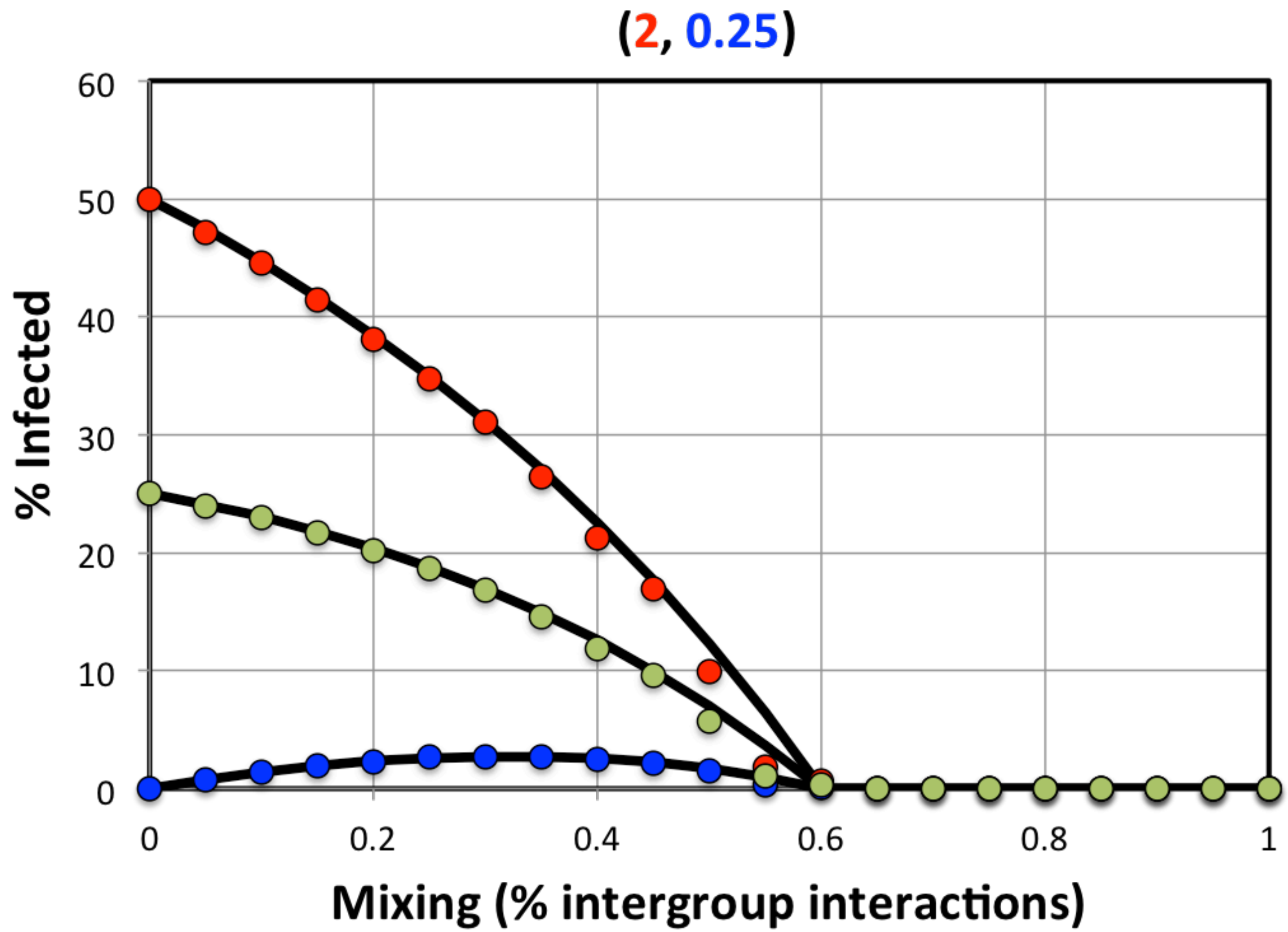
Robustness

- More general model
  - P(Not Infected -> Infected | paired with infected) (v)
  - P(Not Infected -> Infected | paired with Not infected) (0)
  - P(Infected -> Not Infected | paired with infected) (δ)
  - P(Infected -> Not Infected | paired with Not infected) (δ)
- Heterogeneity in agents' susceptibilities

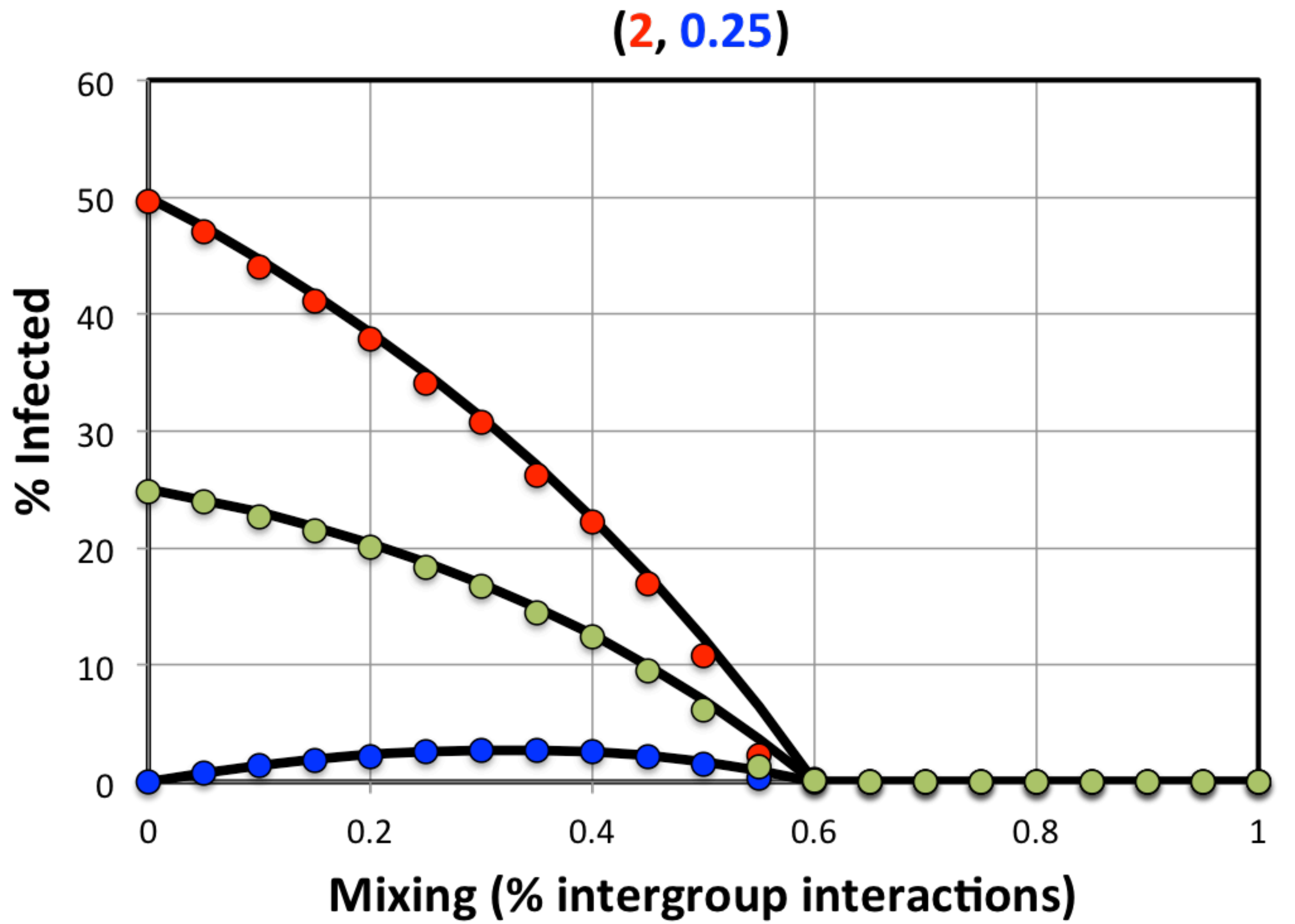
Robustness: variability = 0%



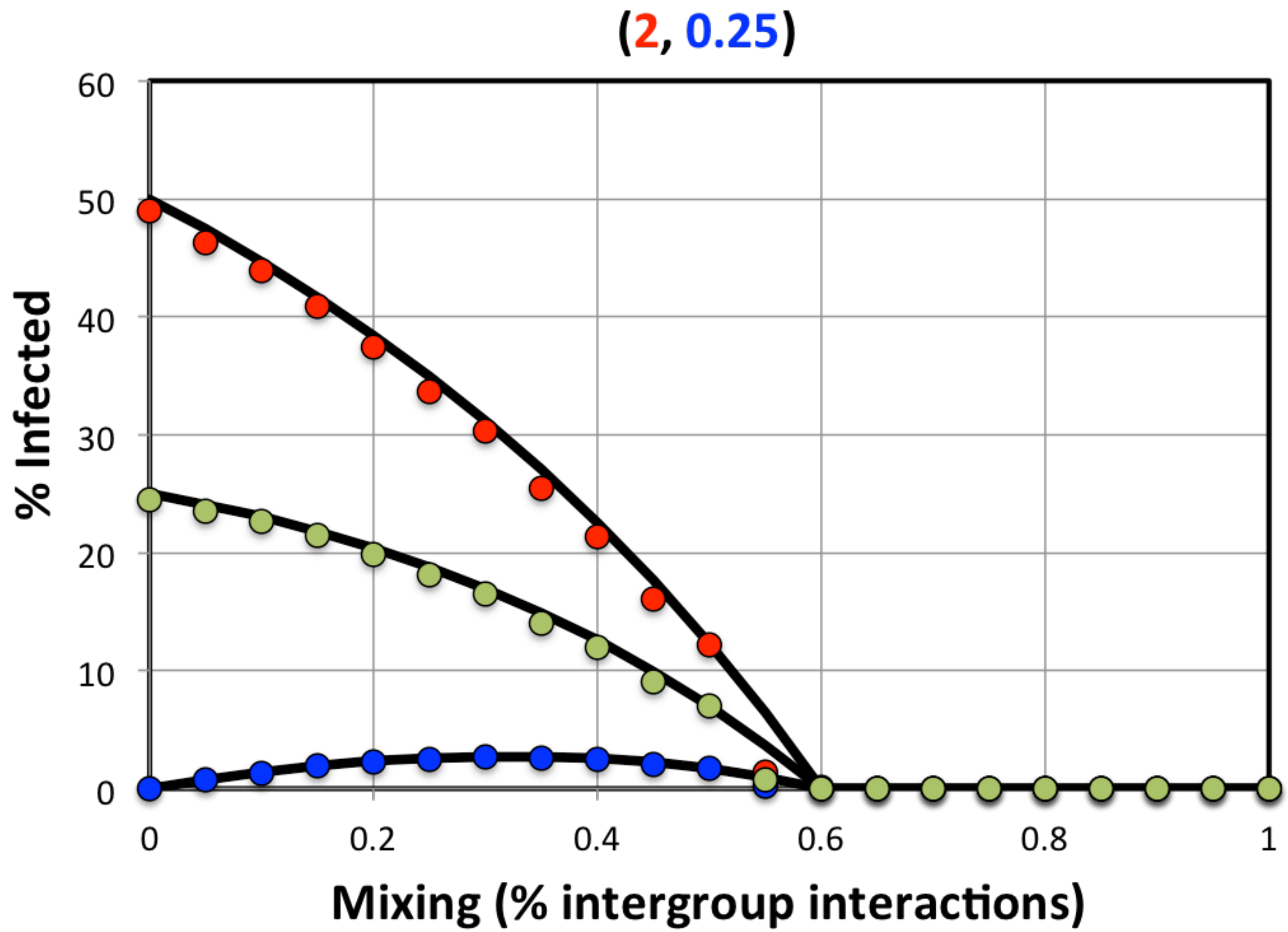
Robustness: variability = 10%



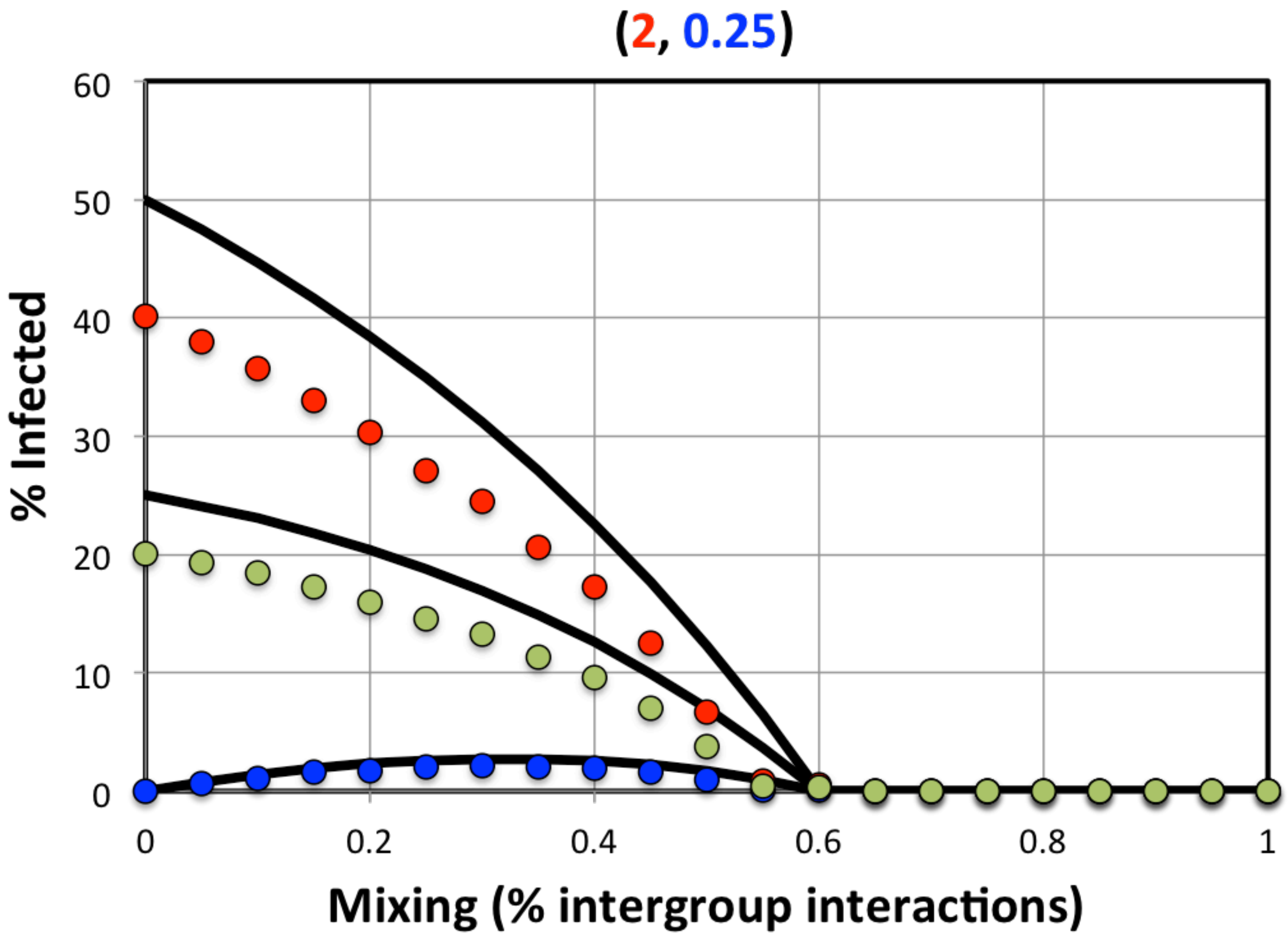
Robustness: variability = 20%



Robustness: variability = 30%



Robustness: variability = 100%



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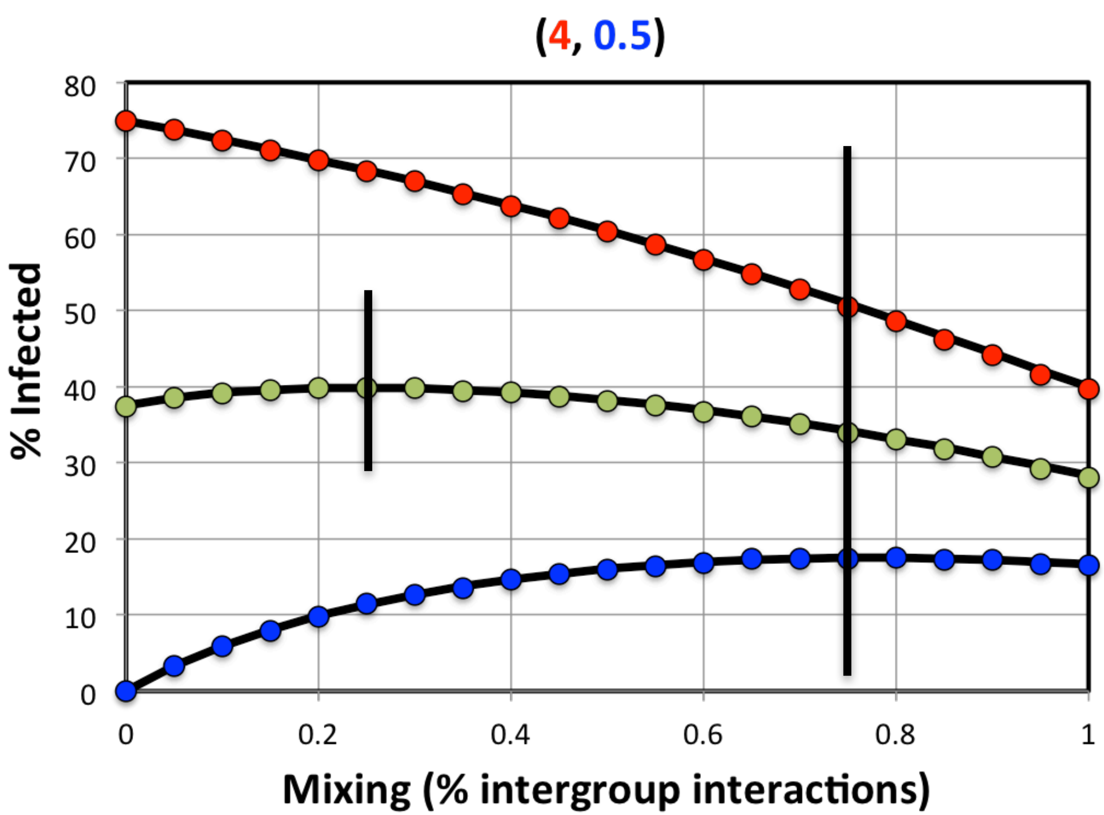
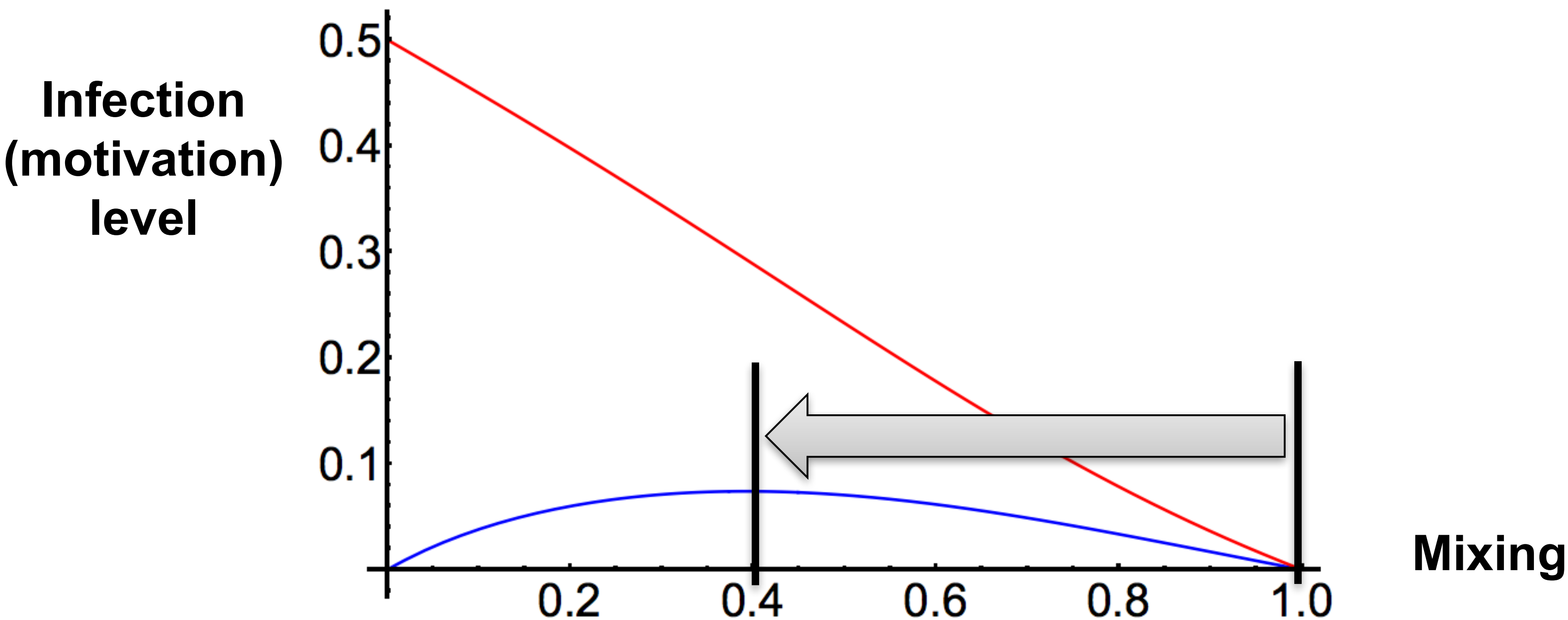
Conclusions

- Overall optimum levels of mixing (or segregation) may not be at the extremes (for the resistant group and for the whole group). Some levels of segregation are not Pareto optimal.



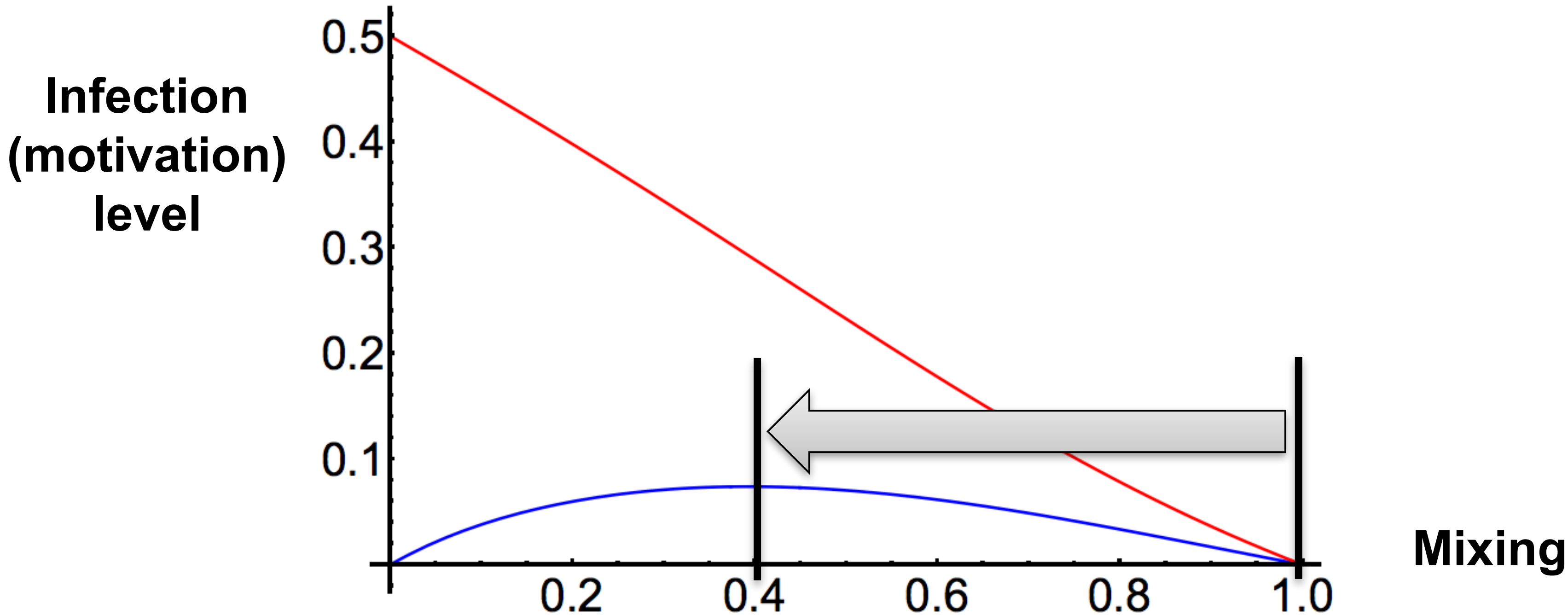
Conclusions

- <u>School case</u>: It may well be in the harder-to-motivate group's interest to protect the easier-to-motivate group from interacting too much with them.



Conclusions

- School case: Policies which assume that, in order to improve the performance of lower-performance students, it will always be good to increase their mixing with higher-performance students, can be misguided. **Increasing segregation may benefit everyone.**



Conclusions

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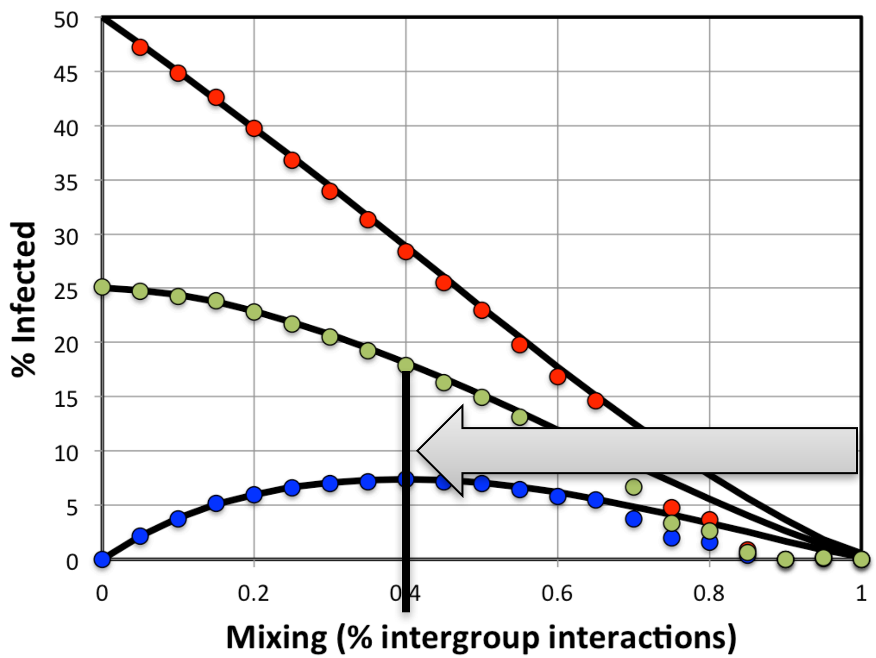
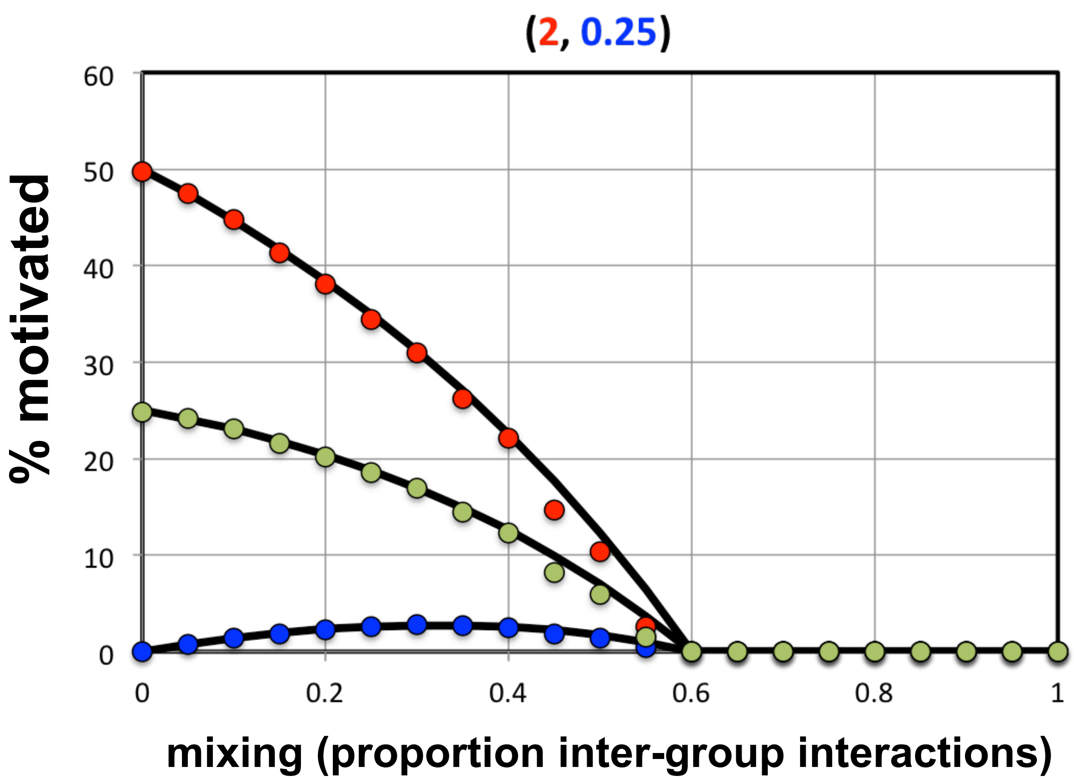
Infection (motivation) level



Mixing

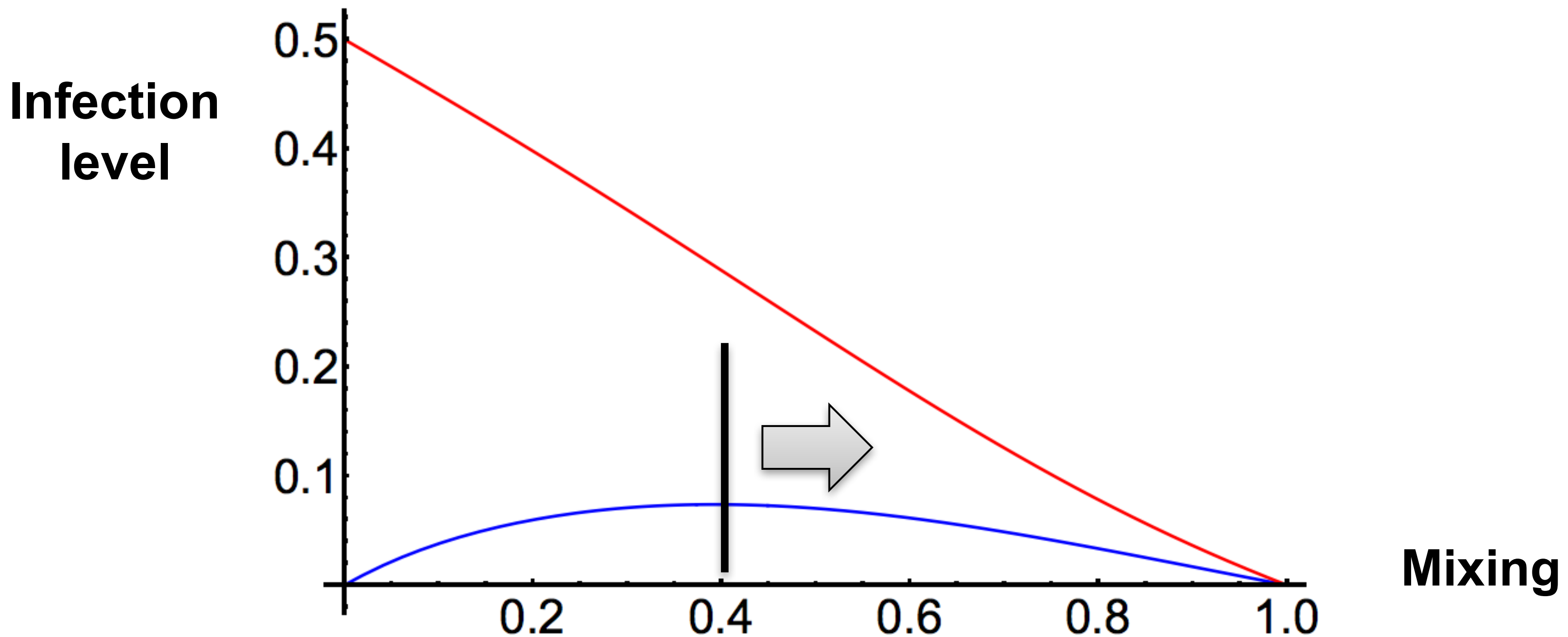
Conclusions

- School case: In extreme cases, the harder-to-motivate group may “kill” everyone’s motivation if they interact too much with the easier-to-motivate group.



Conclusions

- <u>Disease case</u>: Increasing your mixing with a group that has a greater level of infection than you can be beneficial for you (and, naturally, for them).



Conclusions

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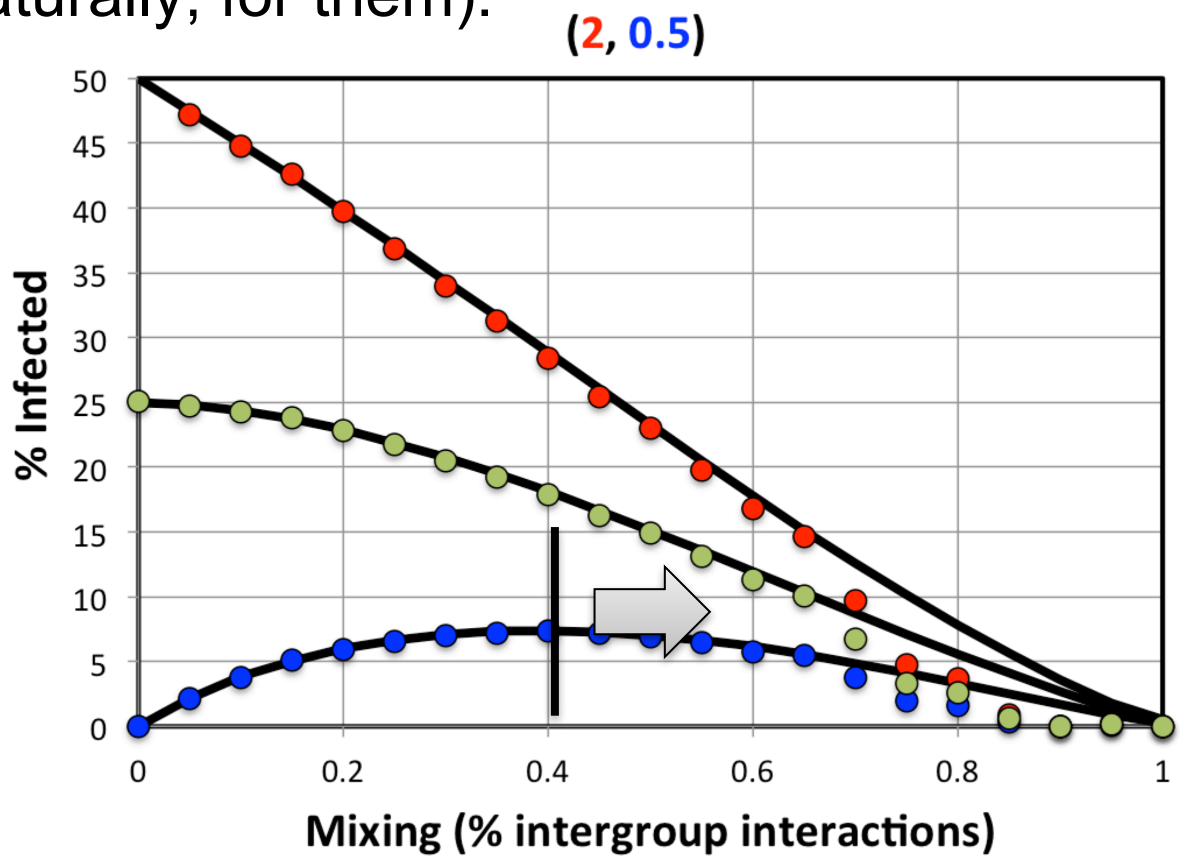

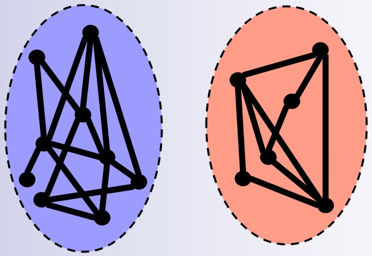
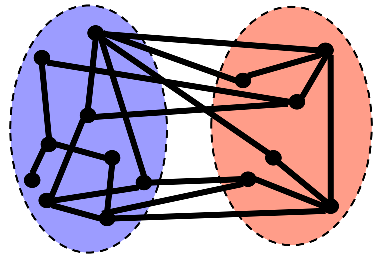
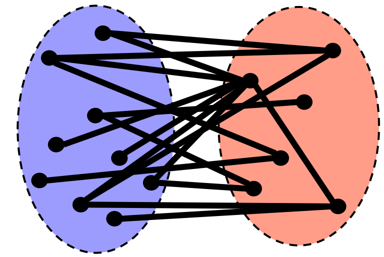

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