



# Should I stay or should I go?

Conditional dissociation in  
the Evolutionary Emergence of Cooperation

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# Outline

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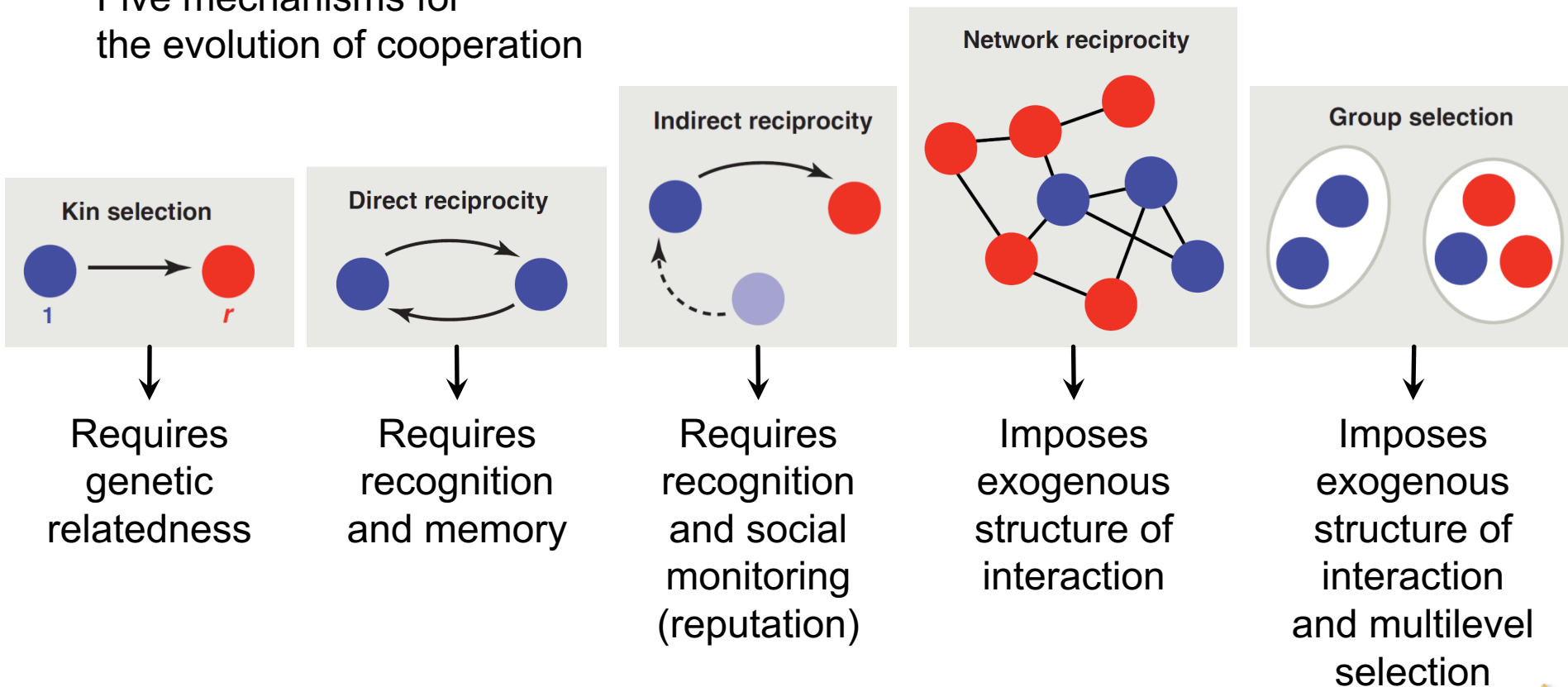
- Introduction
- The question and the approach
- The model
- Results
- A mean-field approximation
- Conclusions



# Introduction

## How did cooperative behaviour evolve?

Five mechanisms for the evolution of cooperation



● Cooperators ● Defectors

Nowak, M. A. (2006) Five rules for the evolution of cooperation. *Science* 314, 1560-1563.



# Introduction

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## Interested in **conditional assortment**

Situations where individuals have an influence in determining with whom they interact (or not)

- ***Pre-interaction*** partner selection (or refusal)
  - Requires advanced cognitive abilities.
  - E.g.: Indirect reciprocity (reputation), external signals or tags...
- ***Post-interaction*** partner-refusal
  - It does not require memory or the ability to anticipate the behaviour of new partners.
  - It only requires the capacity to escape an unpleasant situation.



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# The question

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- Can a simple mechanism of **post-interaction partner refusal** (i.e. **conditional dissociation**) explain the evolutionary emergence of cooperation among unrelated individuals?



# The approach

## Game Theory as a Framework

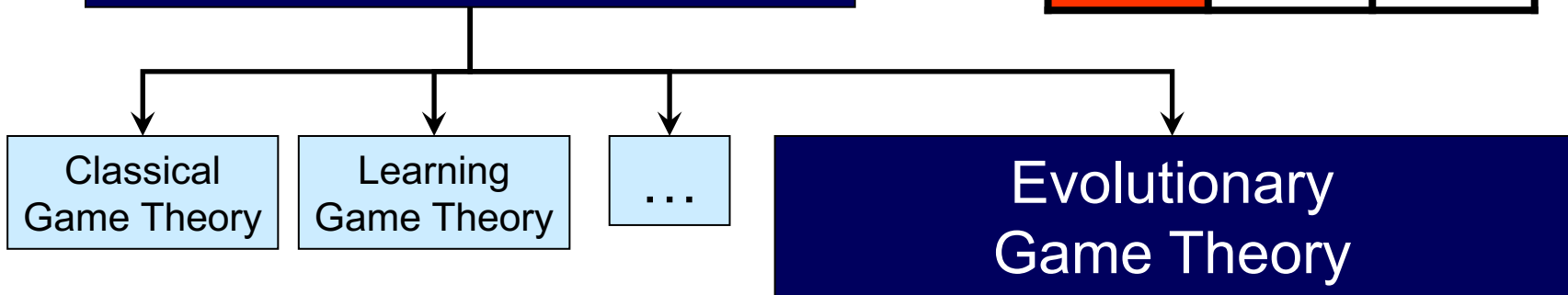
Player 2

		Cooperate	Defect
Player 1	Cooperate	3, 3	0, 4
	Defect	4, 0	1, 1



# The approach

## Game Theory as a Framework



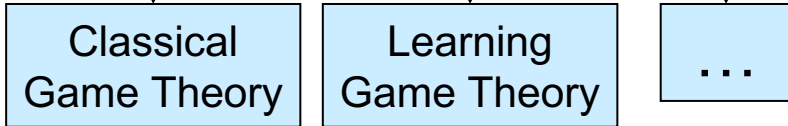
		Player 2	
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	Defect	4, 0	1, 1



# The approach

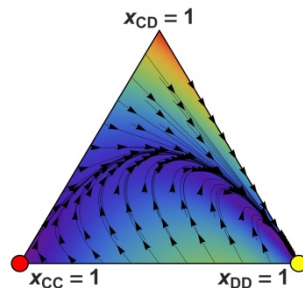
## Game Theory as a Framework

		Player 2	
		Cooperate	Defect
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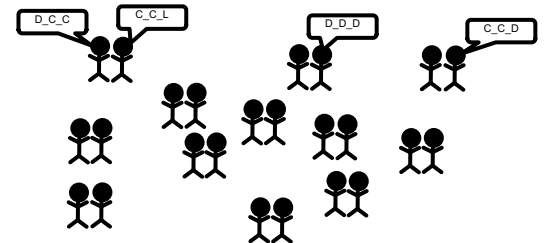


## Evolutionary Game Theory

### Mainstream Evolutionary Game Theory



### Explicit modelling of actual populations

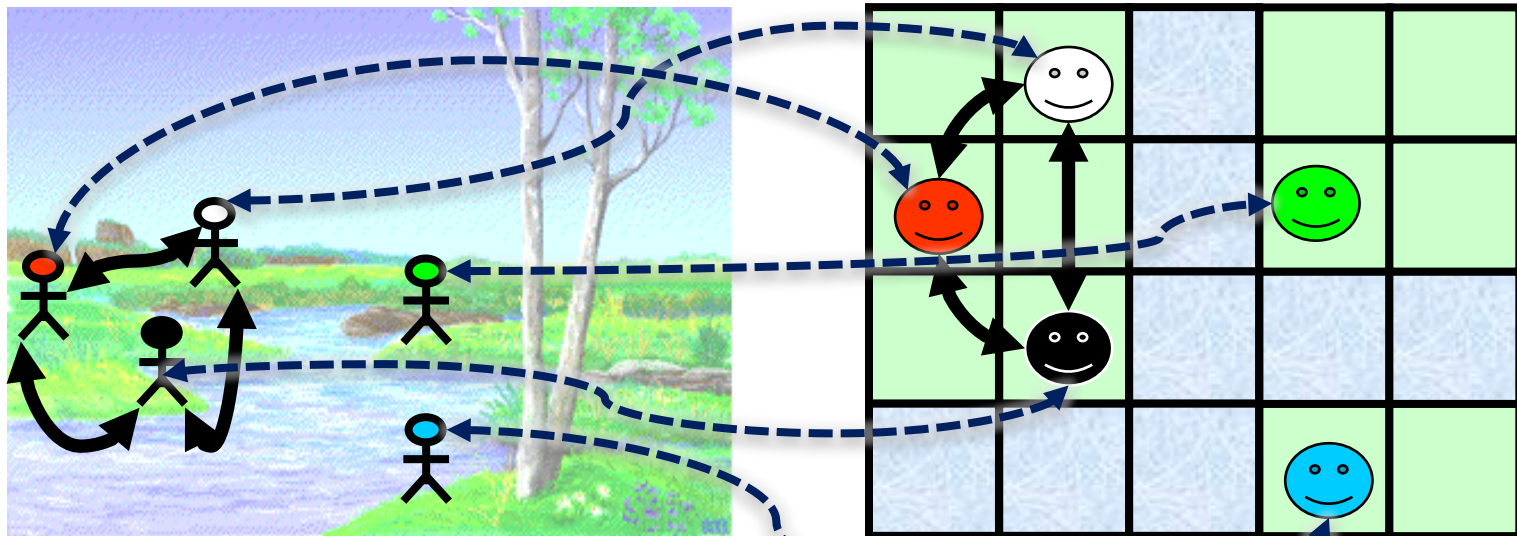


# The approach

- Agent-based modelling:

Target (real) system

Agent-based model



Entities



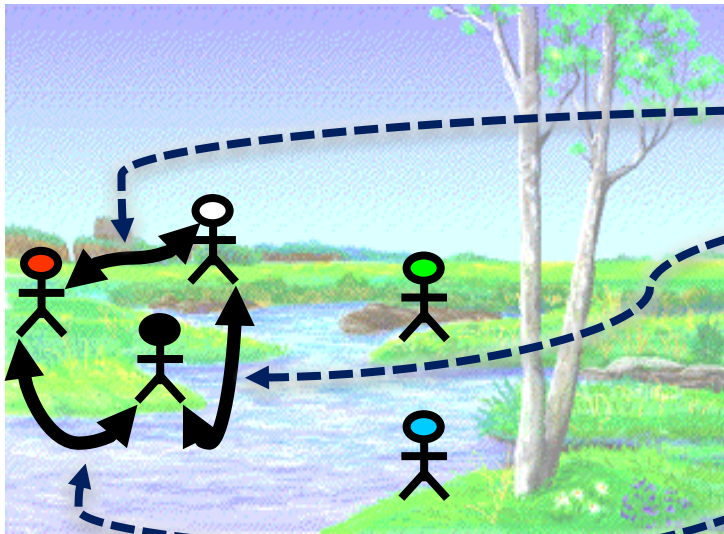
Agents



# The approach

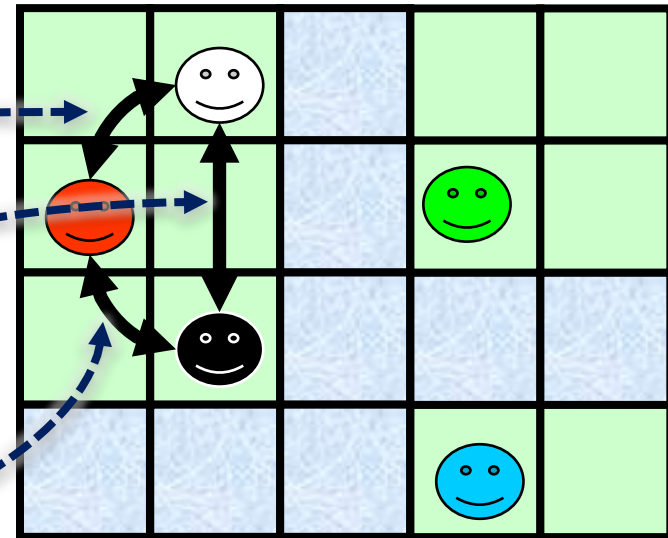
- Agent-based modelling:

Target (real) system



Interactions among entities

Agent-based model



Interactions among agents



# The approach

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- Selection of strategies:

All individuals in the population share a common capability to gather information and to condition their actions on that information, and then allow for *every possible strategy within such a uniform setup*.

- Beyond the identification of “winning” strategies.

Also concerned with the issue of whether some well-defined *pattern of aggregate behaviour* eventually emerges.

(Even under perpetual turnover in strategy choice)

- In short, we strive to understand what factors determine the emergence and stability of cooperative behaviour.



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# The game (Prisoner's Dilemma)

The Prisoner's Dilemma:  
Payoffs

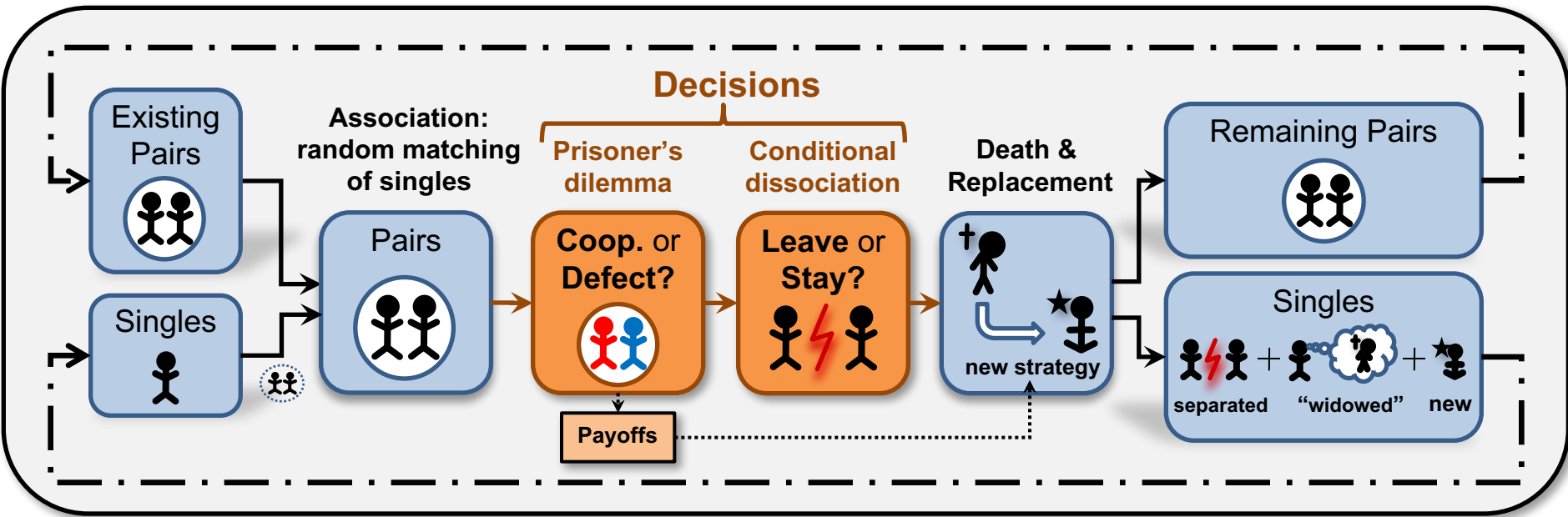
Player 1

Player 2

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3	0, 4
	Defect	4, 0	1, 1



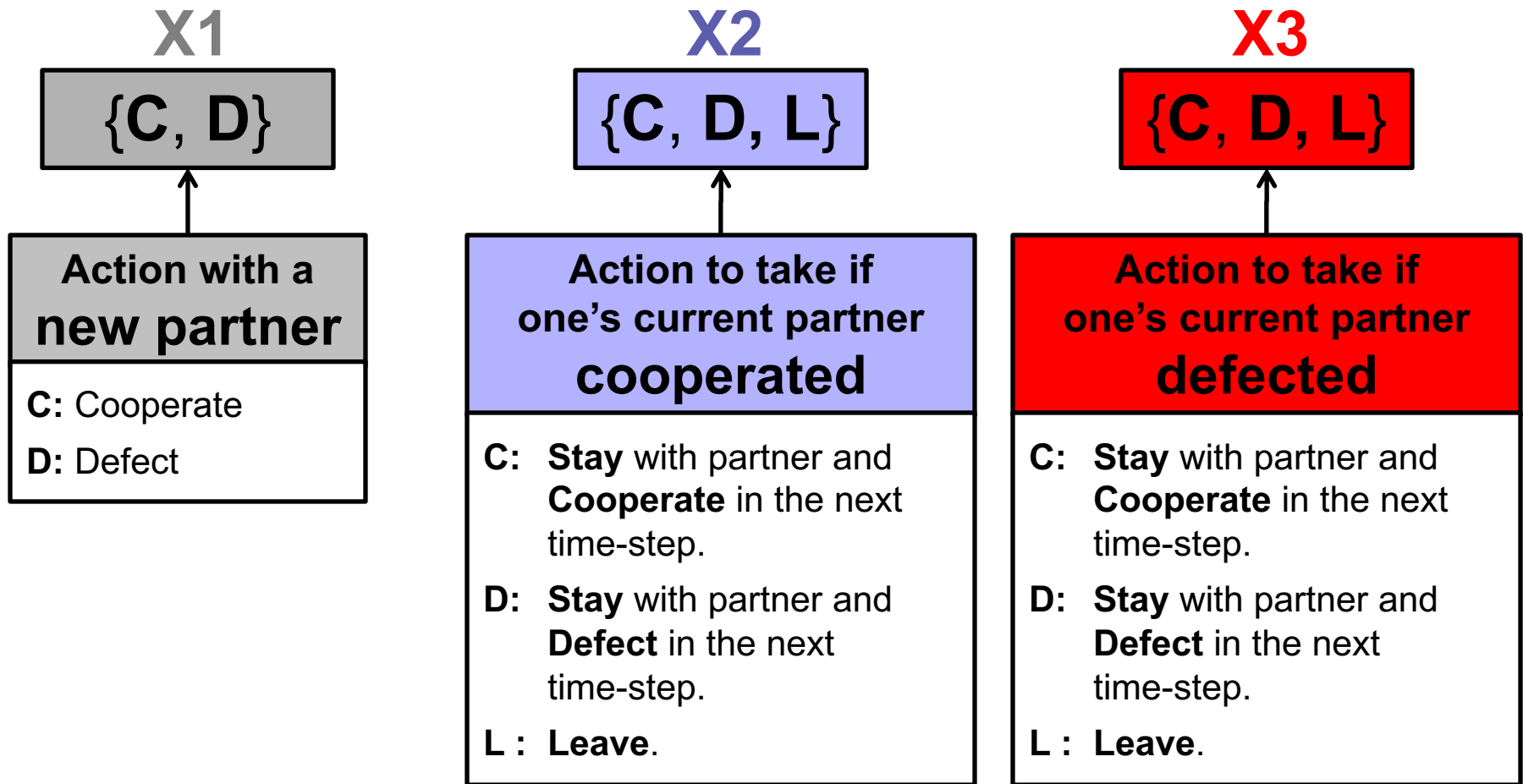
# The timeline



Sequence of events within each time period



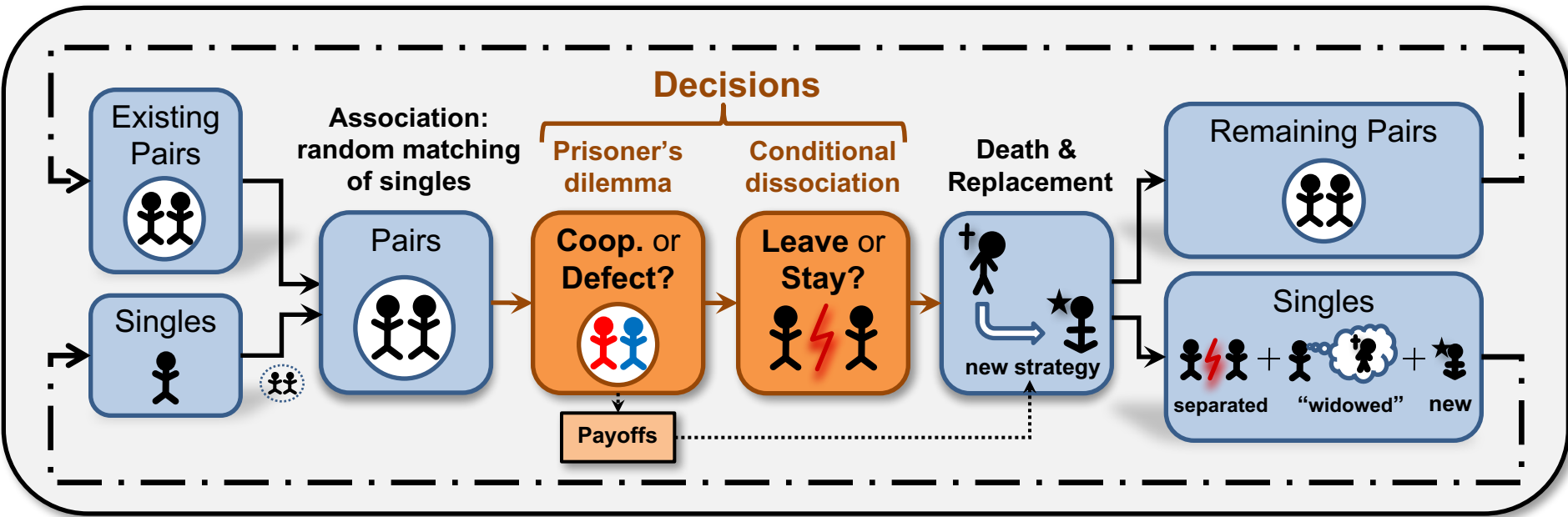
# The strategies: X1\_X2\_X3



Cooperate and leave after a partner's defection: C\_C\_L



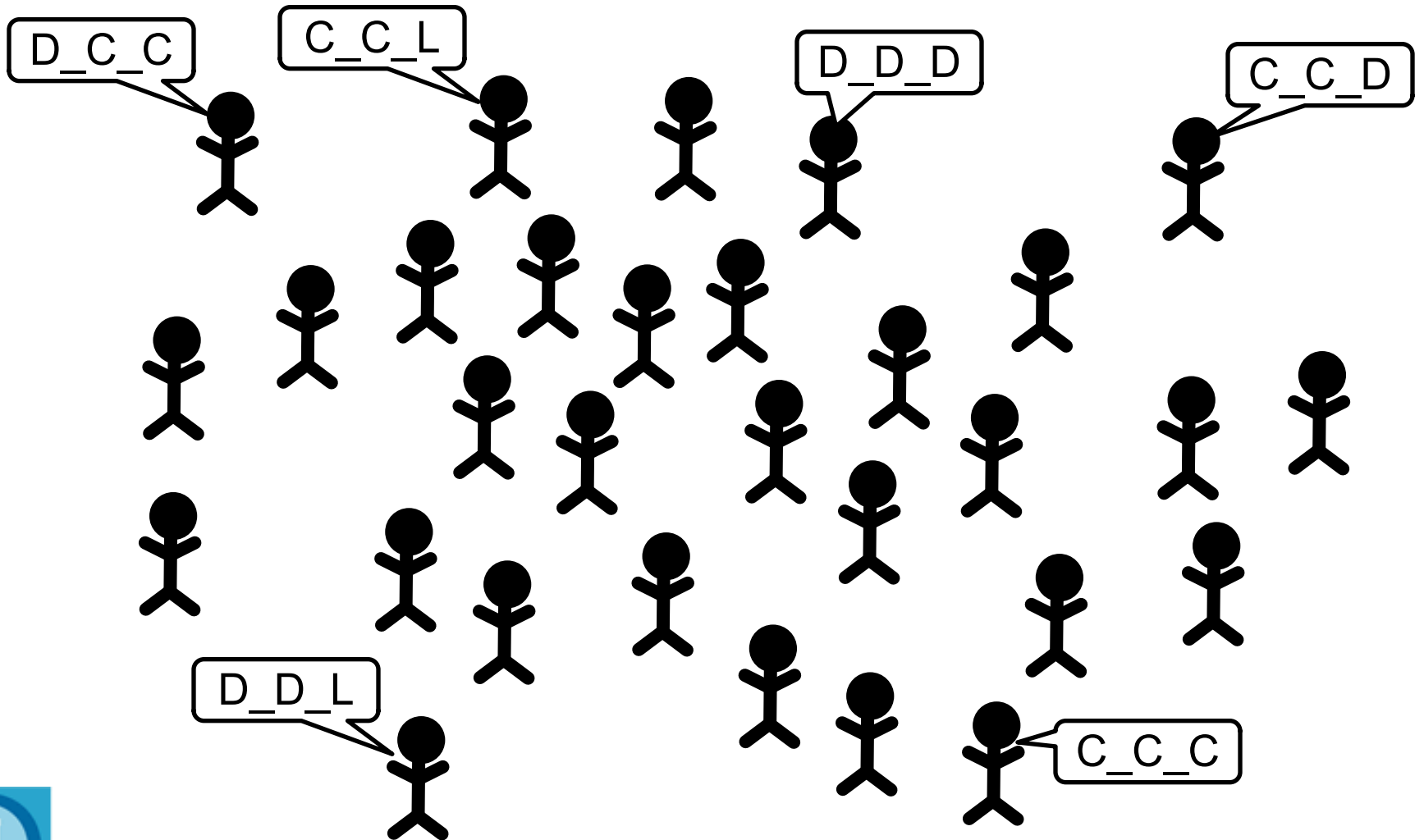
# The timeline



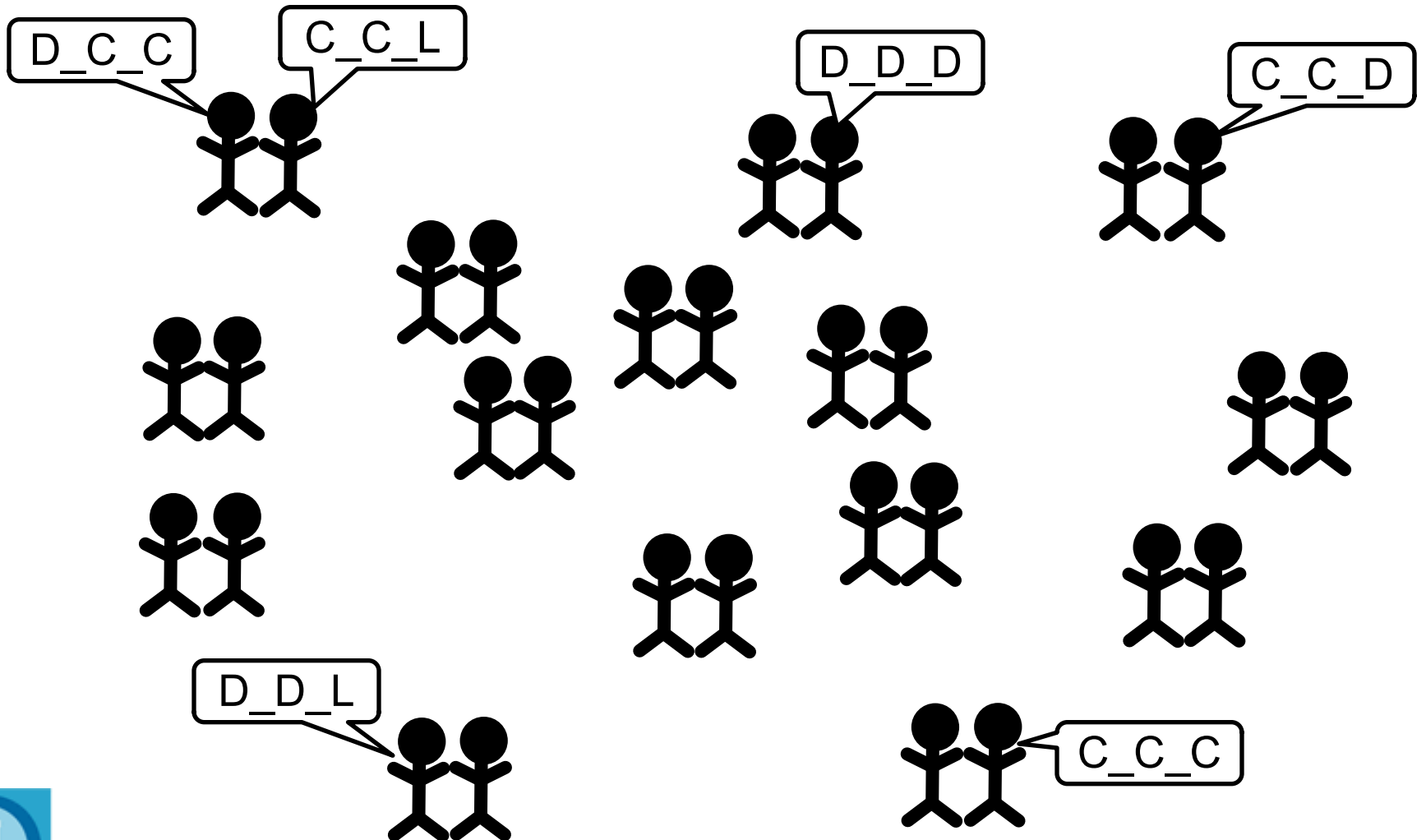
Sequence of events within each time period



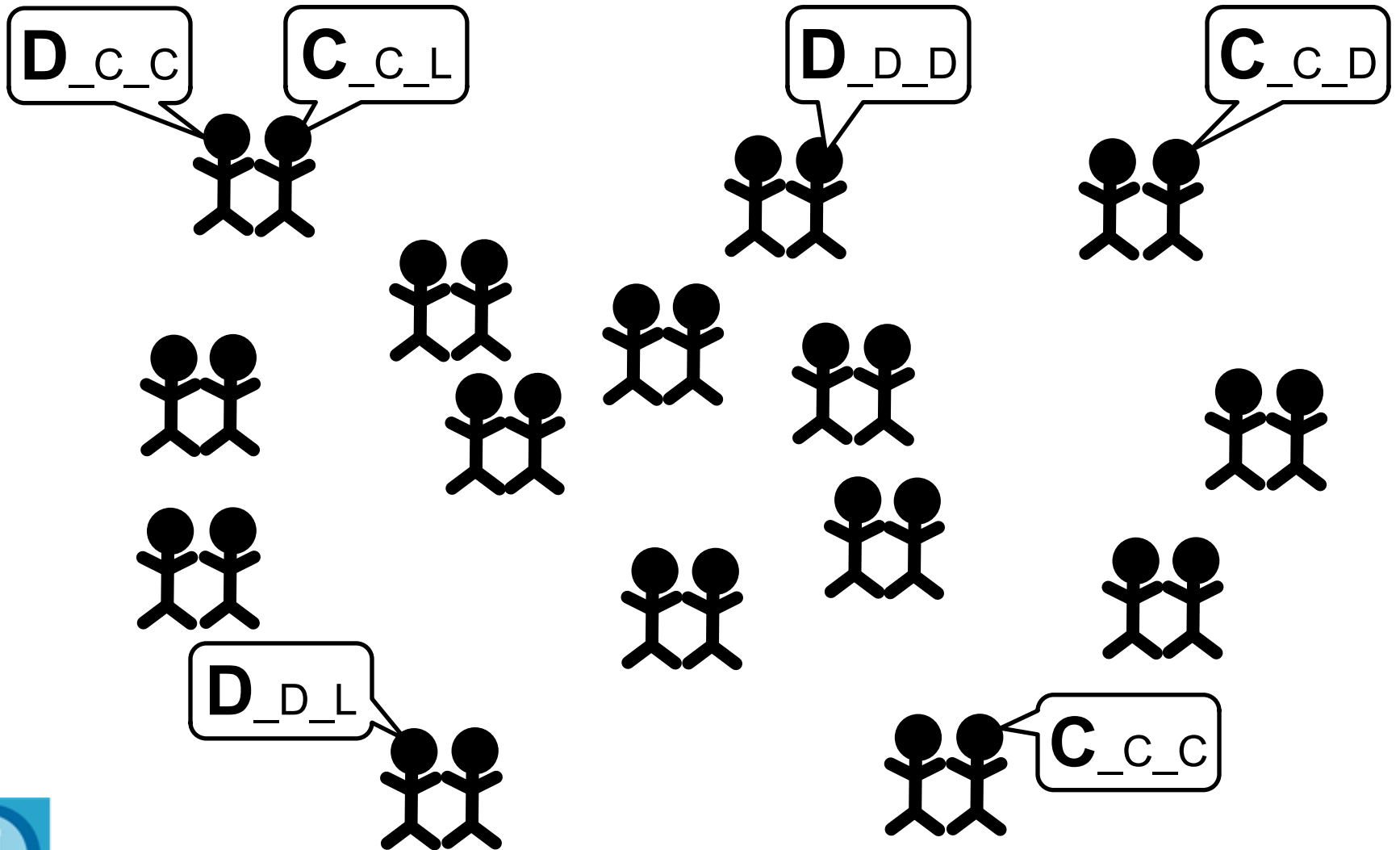
# The initial population



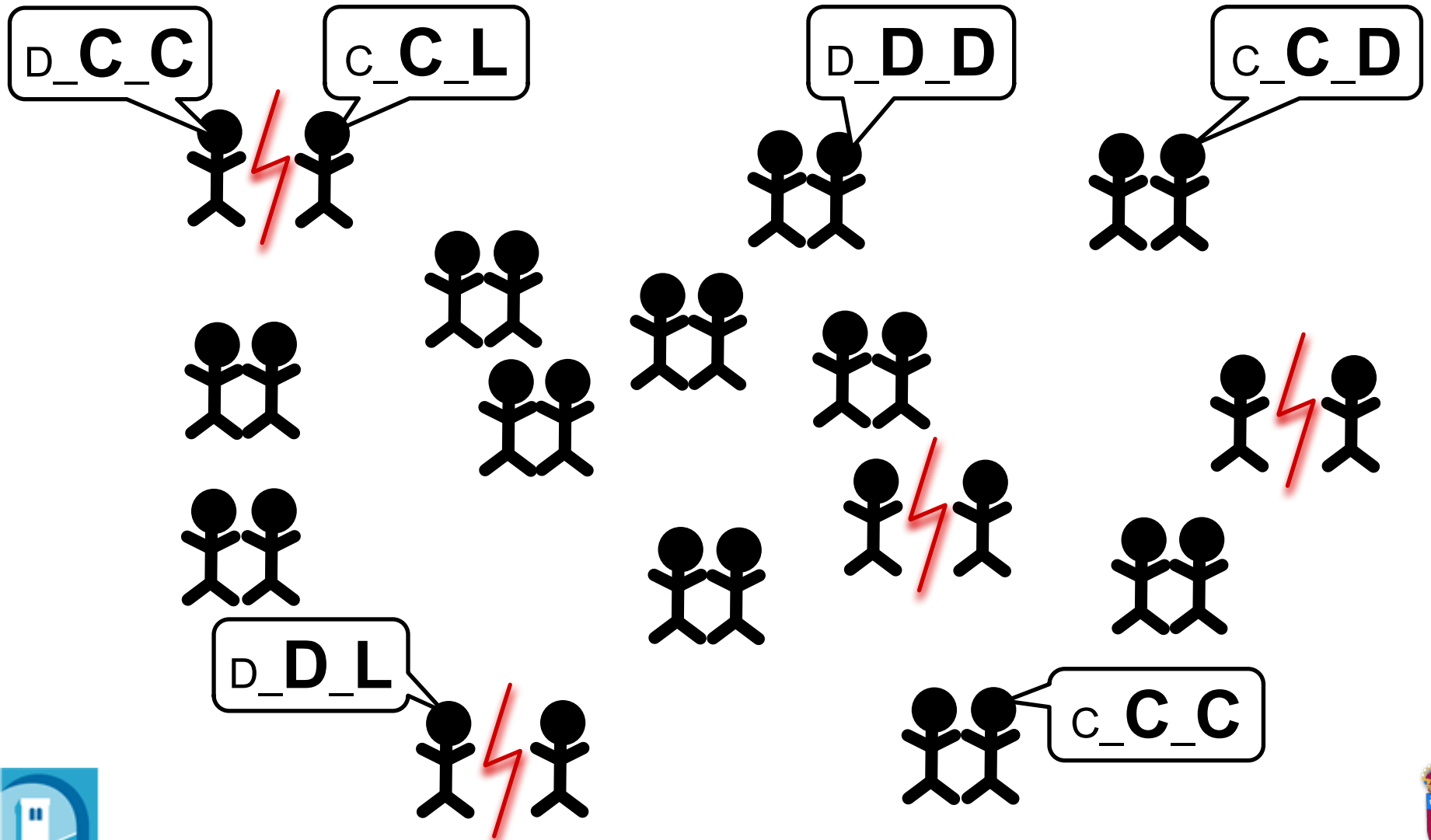
# The random pairing of singles



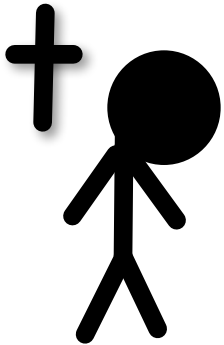
# The Prisoner's Dilemma game



# Should I stay or should I go?



# Death and Replacement



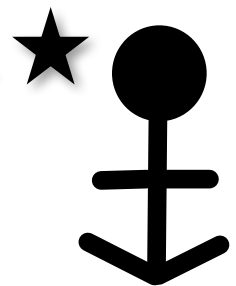
Individuals randomly die, an event that occurs independently for each individual with probability  $p$  in each time period.

Hence, **the lifespan of an individual is geometrically distributed with mean-value *expLife* =  $1/p$ .**

Each dead individual is immediately replaced by a new entrant

Reproduction is proportional to payoffs and population size is kept constant.

(The probability of adopting a certain strategy is proportional to its aggregate payoff).

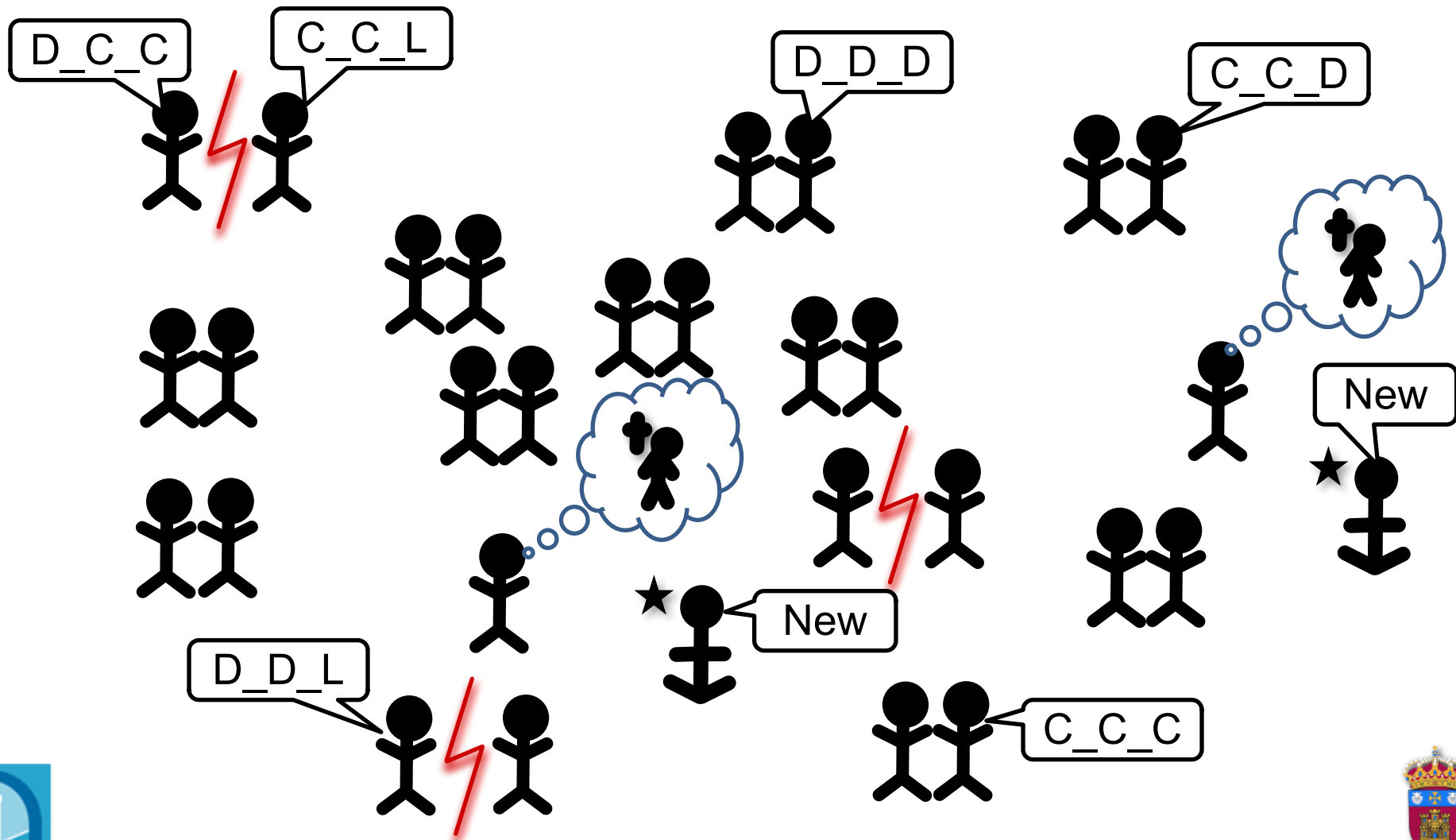


**new strategy**

**Random mutations** occur with probability  $\mu$ .

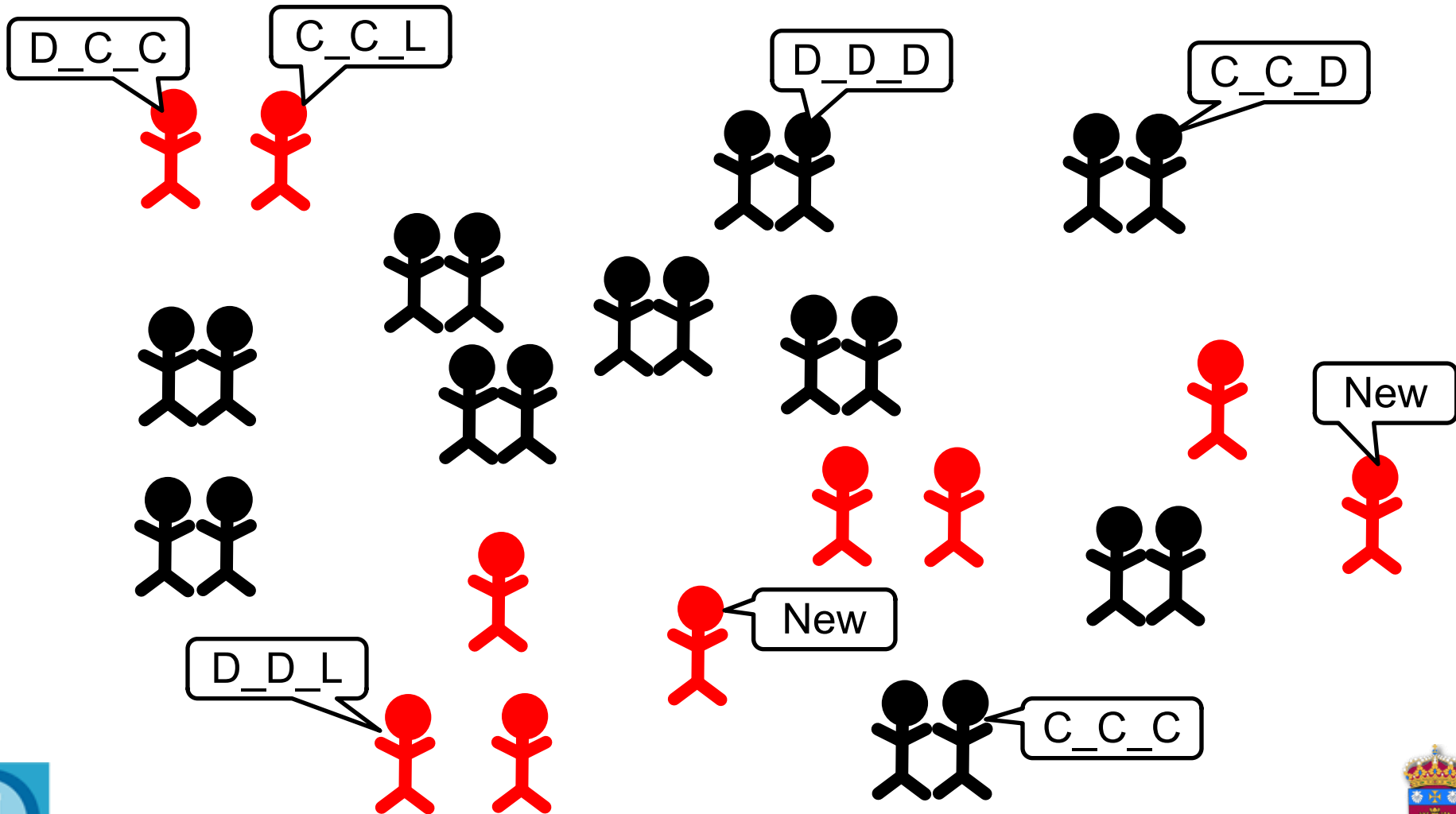


# The population for the next step

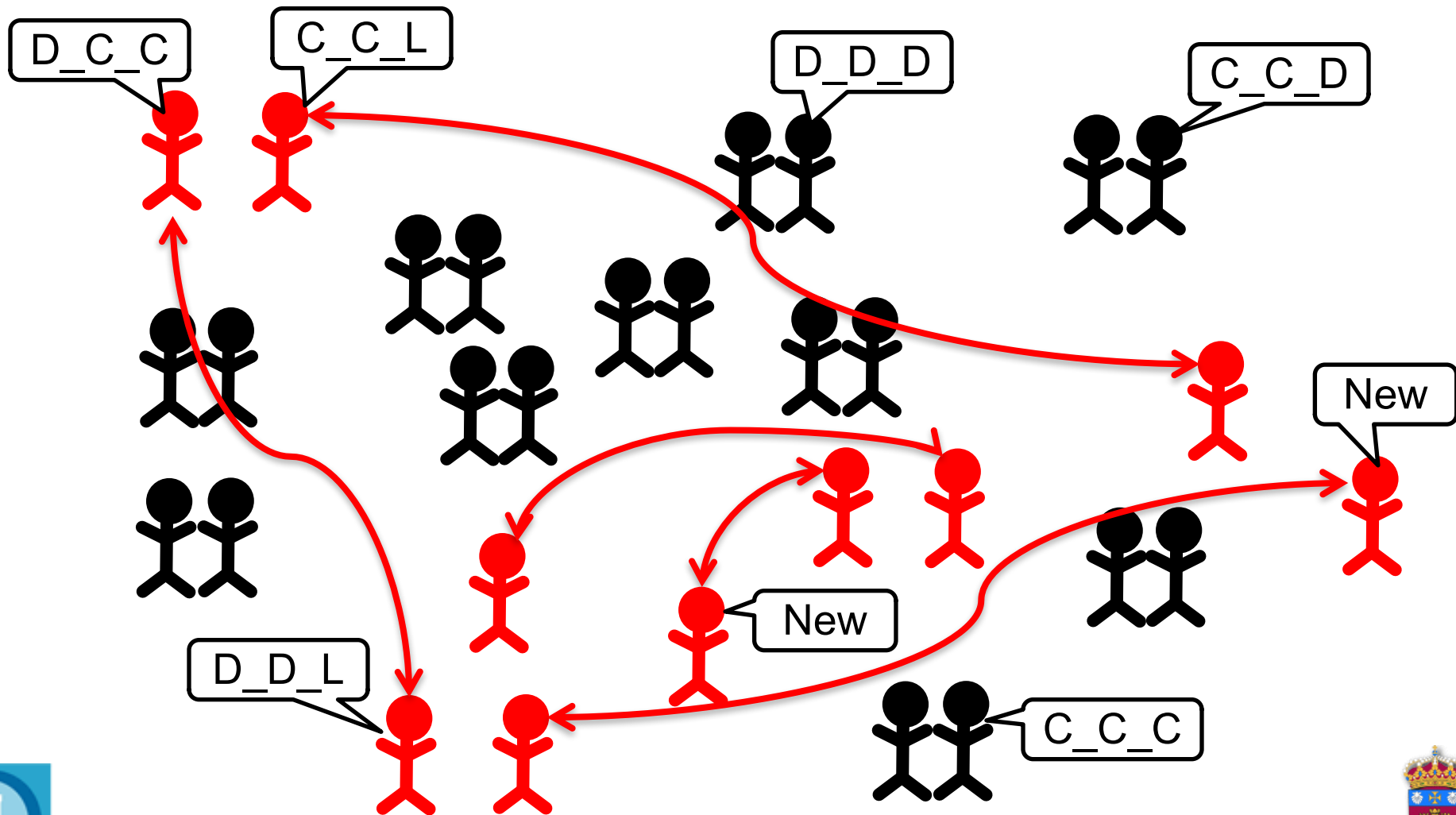




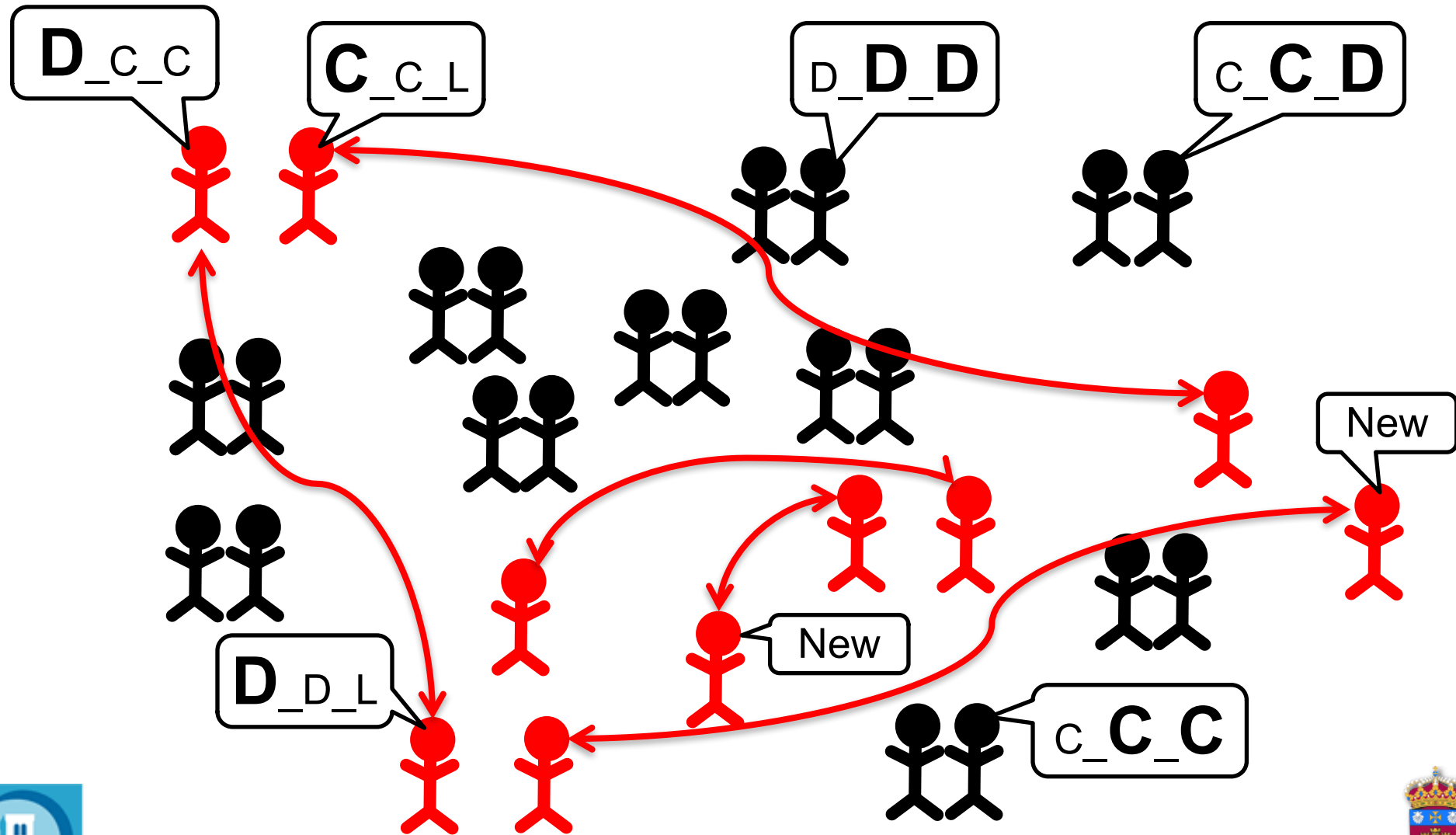
# The random pairing of singles



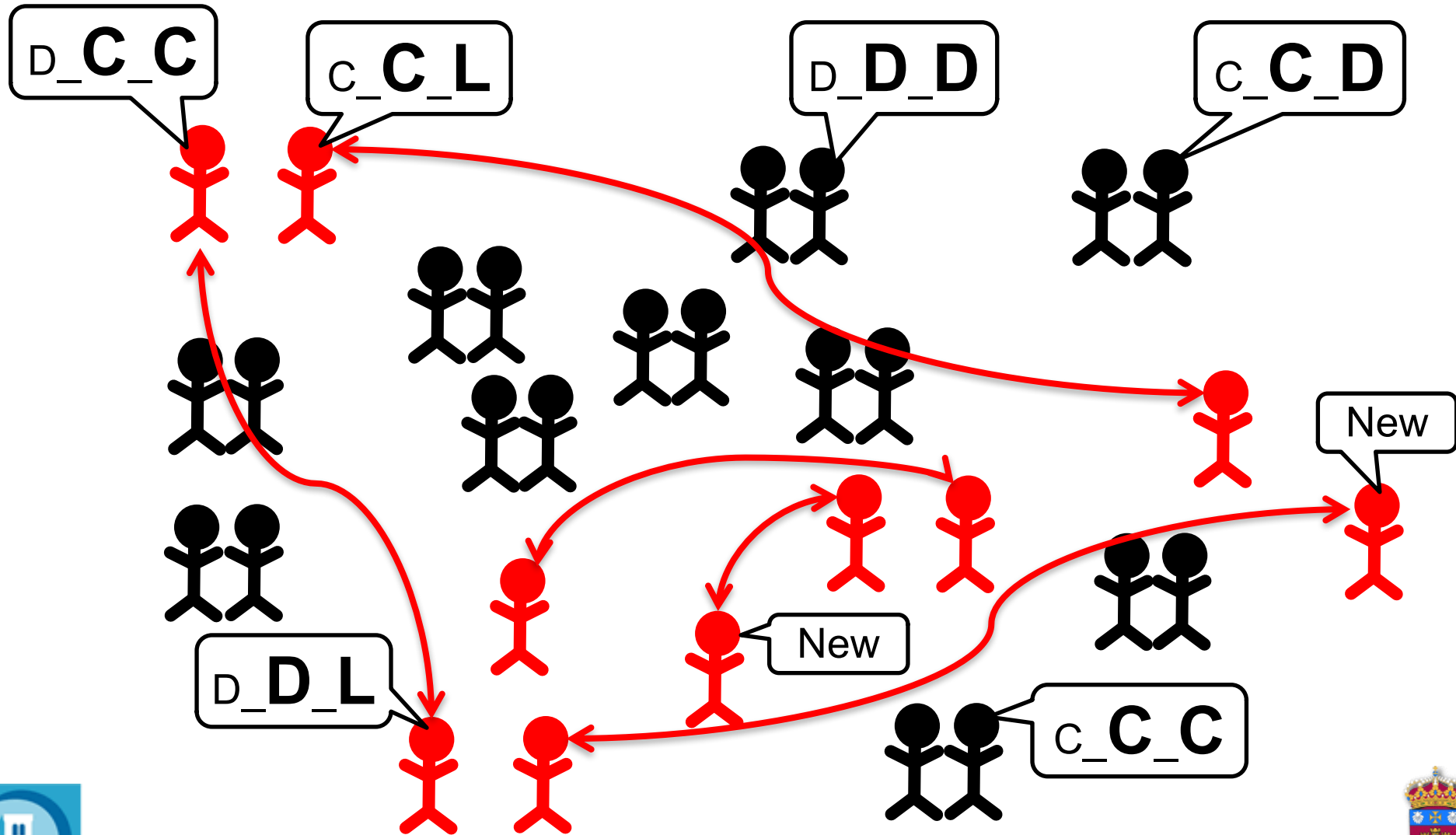
# The random pairing of singles



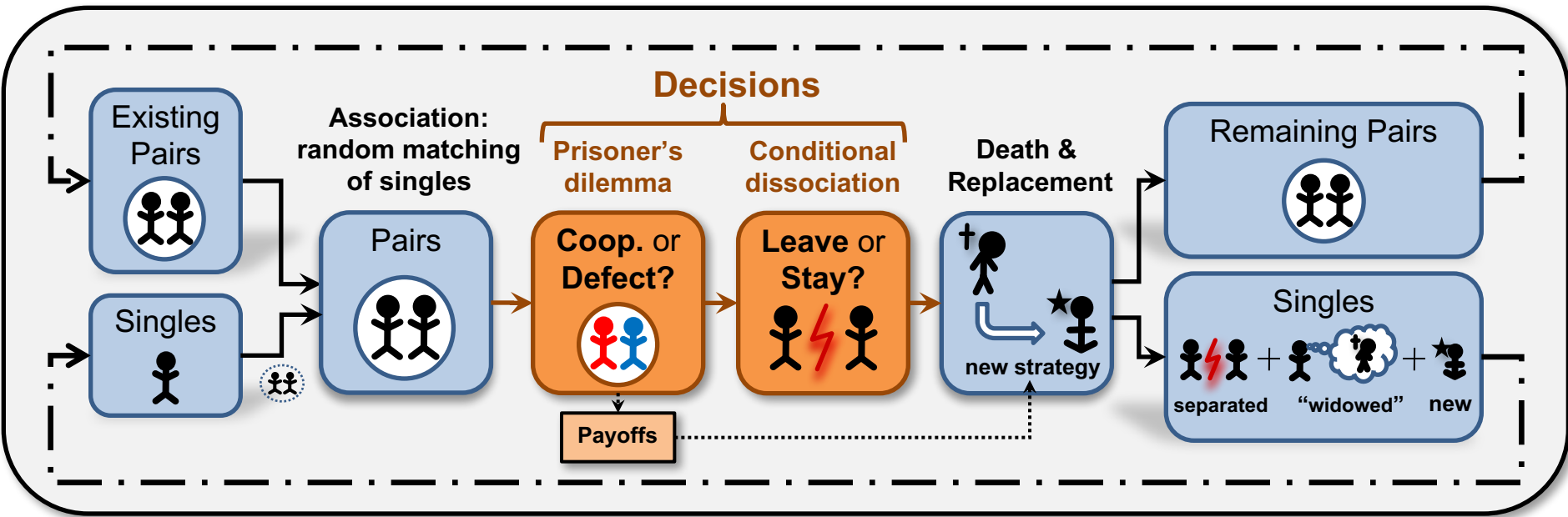
# The Prisoner's Dilemma game



# Should I stay or should I go?



# The timeline



Sequence of events within each time period



# The implementation of the model

Payoffs



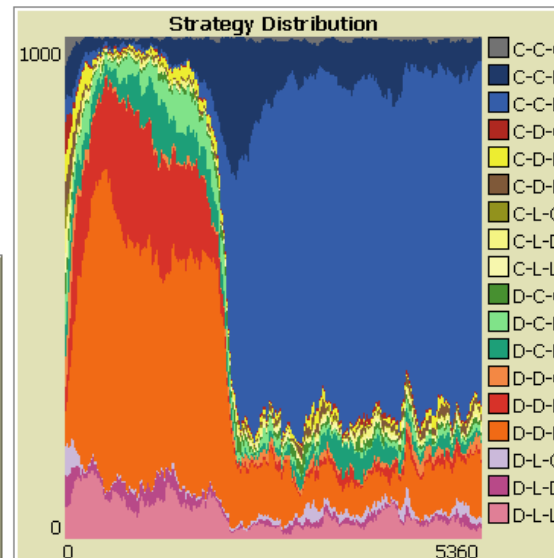
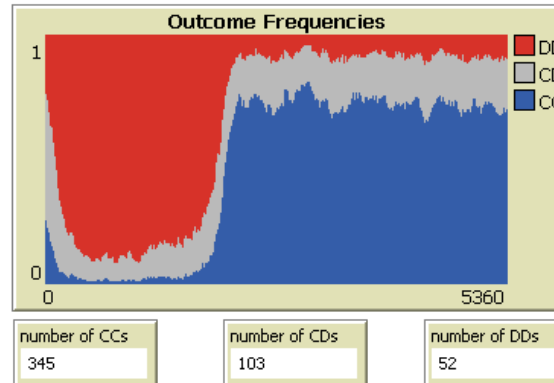
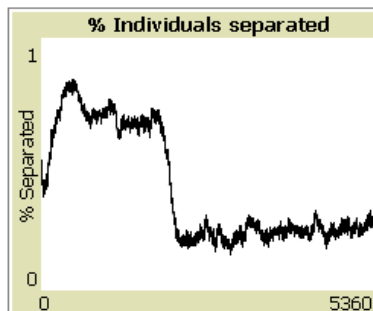
initial-strategy  
random

Setup

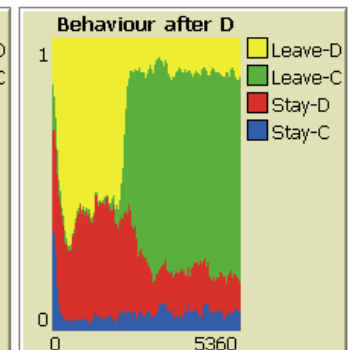
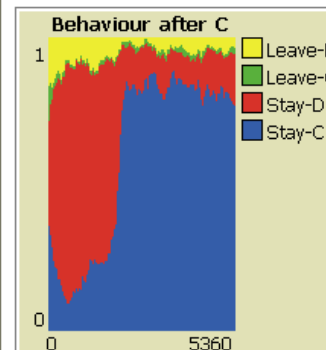
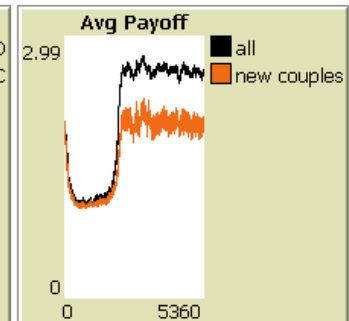
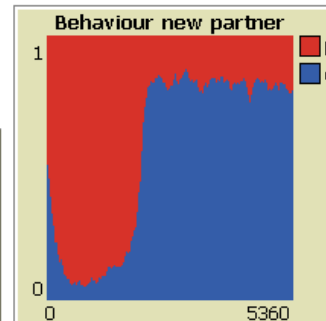
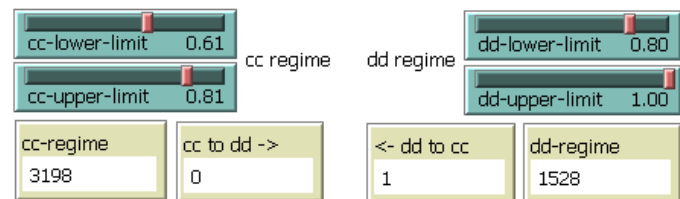
Go once

Go

ticks  
5355



Definition of regimes



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# Three scenarios

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## Key factor:

Expected lifespan of the individuals:

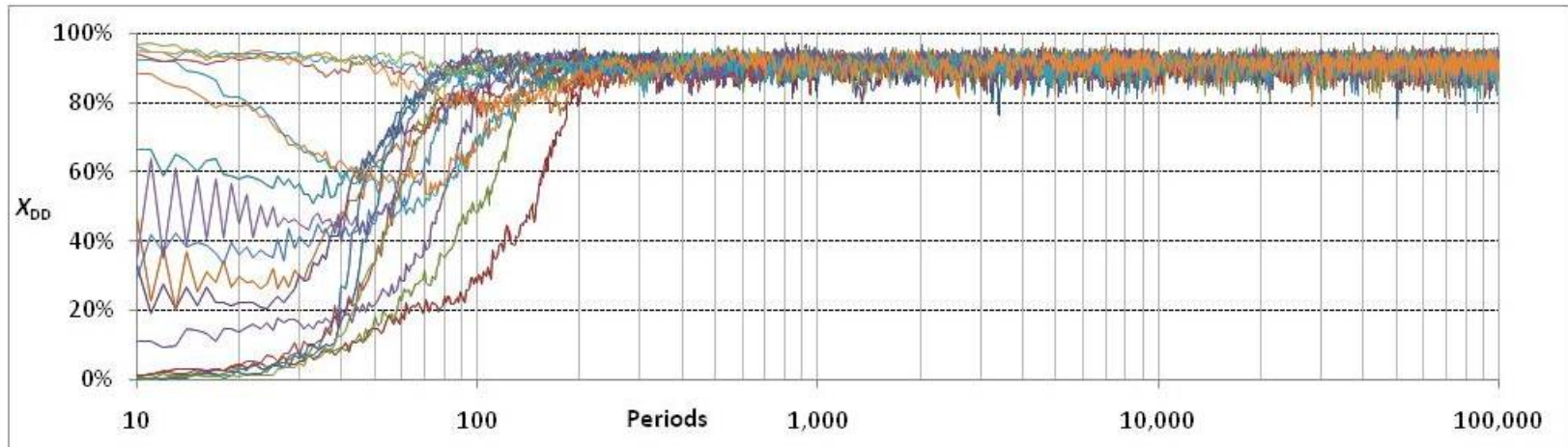
- Short life  $expLife = 5$
- Long life  $expLife = 100$
- Intermediate life  $expLife = 20$

Benchmark:  $N = 1000$ ,  $\mu = 0.05$ ,  $T = 4$ ,  $R = 3$ ,  $P = 1$ ,  $S = 0$ .



# Short life: non-cooperative regime

$x_{DD}$  : share of **DD outcomes** in a period



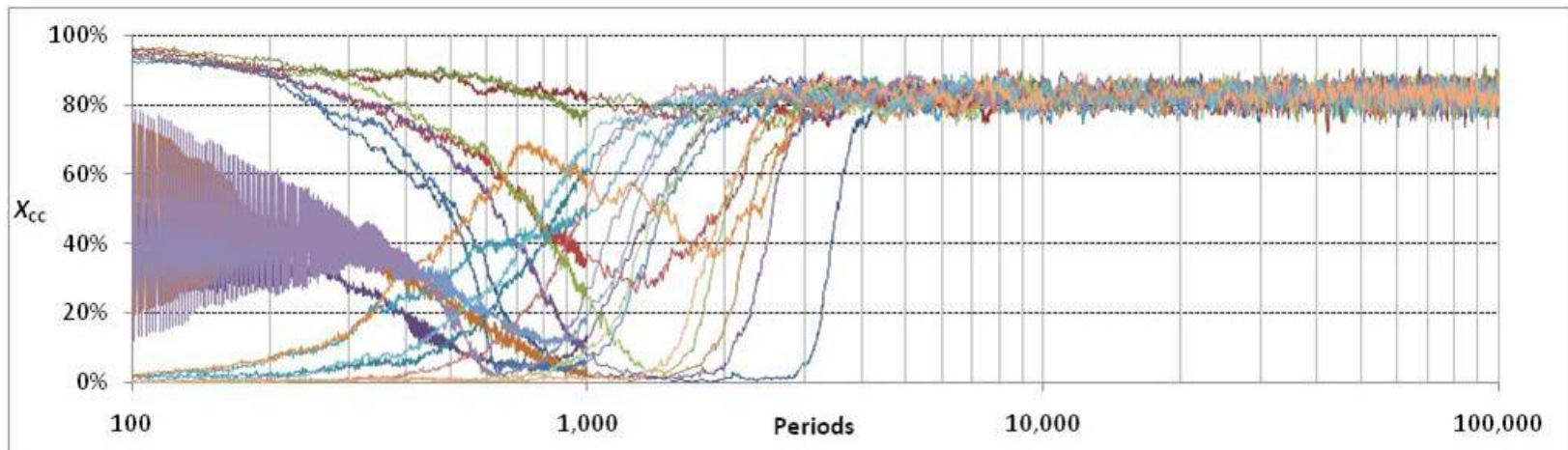
Evolution of the percentage of **DD outcomes** for 18 different runs, each run starting with the whole population using one of the 18 different strategies.

Parameterisation:  $N = 1000$ ,  **$expLife = 5$** ,  $\mu = 0.05$ ,  $T = 4$ ,  $R = 3$ ,  $P = 1$ ,  $S = 0$ .



# Long life: cooperative regime

$x_{CC}$  : share of **CC outcomes** in a period



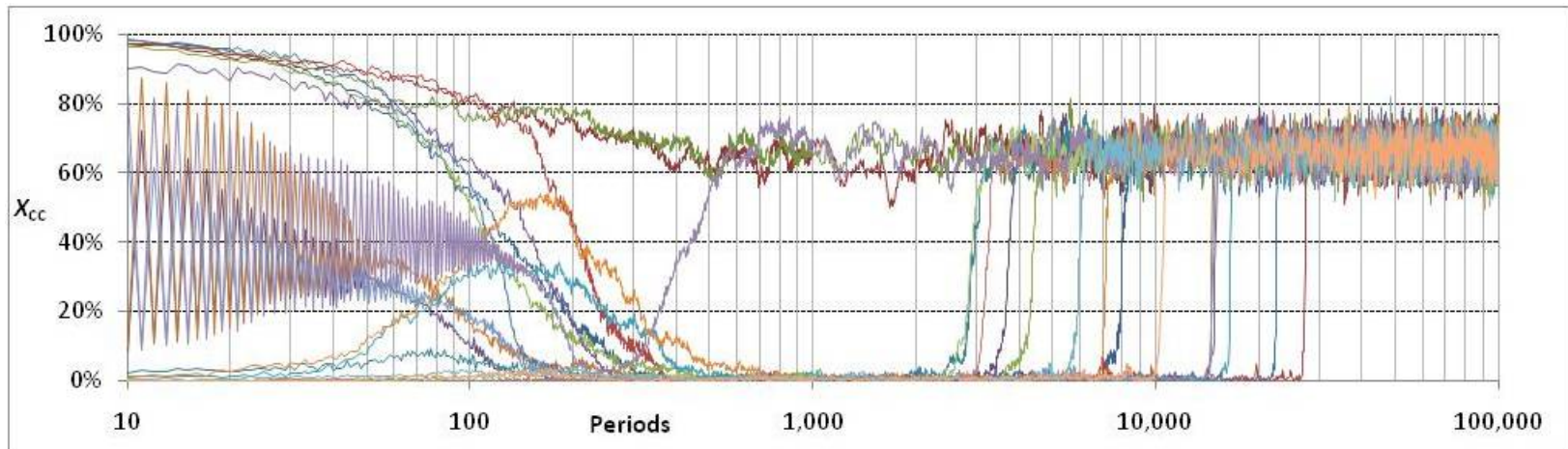
Evolution of the percentage of **CC outcomes** for 18 different runs, each run starting with the whole population using one of the 18 different strategies.

Parameterisation:  $N = 1000$ ,  **$expLife = 100$** ,  $\mu = 0.05$ ,  $T = 4$ ,  $R = 3$ ,  $P = 1$ ,  $S = 0$ .



# Intermediate life: dd-reg -> cc-reg

$x_{CC}$  : share of **CC outcomes** in a period



Evolution of the percentage of **CC outcomes** for 18 different runs, each run starting with the whole population using one of the 18 different strategies.

Parameterisation:  $N = 1000$ , **expLife = 20**,  $\mu = 0.05$ ,  $T = 4$ ,  $R = 3$ ,  $P = 1$ ,  $S = 0$ .



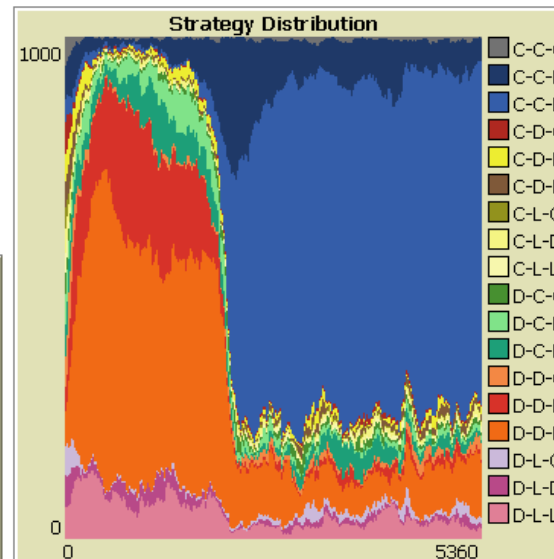
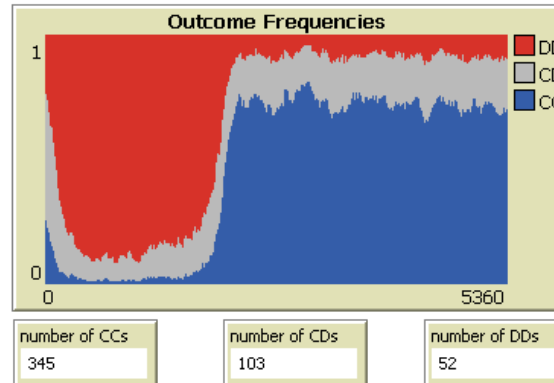
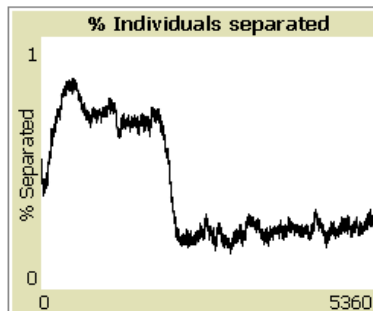
# Intermediate life – The 2 regimes

Payoffs

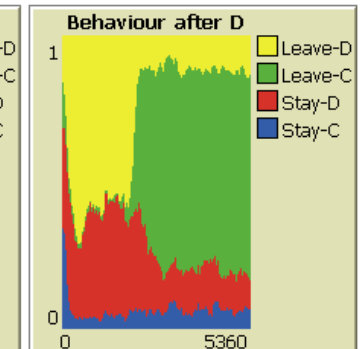
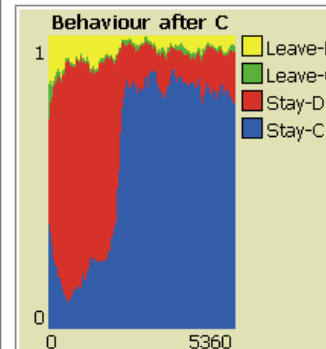
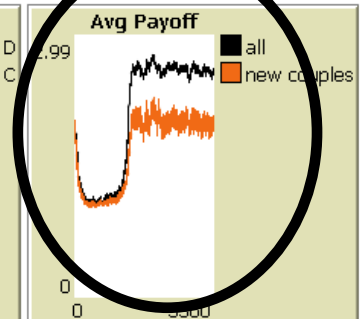
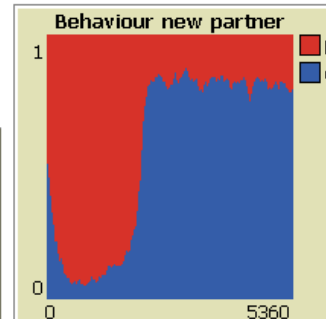
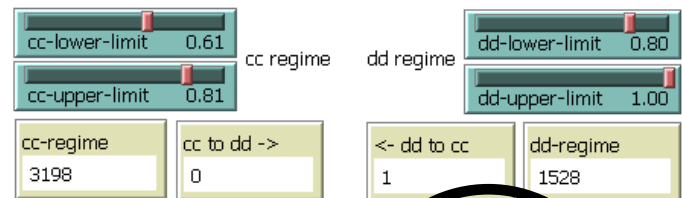


initial-strategy  
random

Setup  
Go once  
Go  
ticks: 5355



Definition of regimes

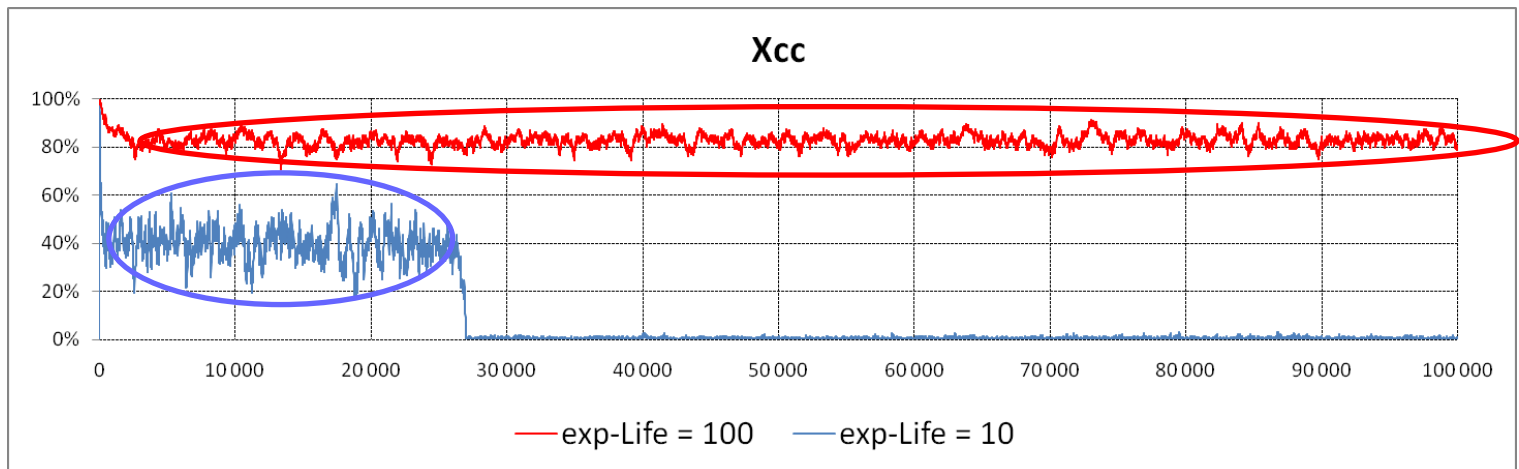


# Characterising the 2 regimes

- Non-cooperative regime... Easy:

$$x_{DD} \geq 0.8$$

- (partially) Cooperative regime... not trivial



Need to define a reference value:  $\hat{x}_{CC}^{CR}(expLife)$

$x_{CC}$  : share of CC outcomes in a period

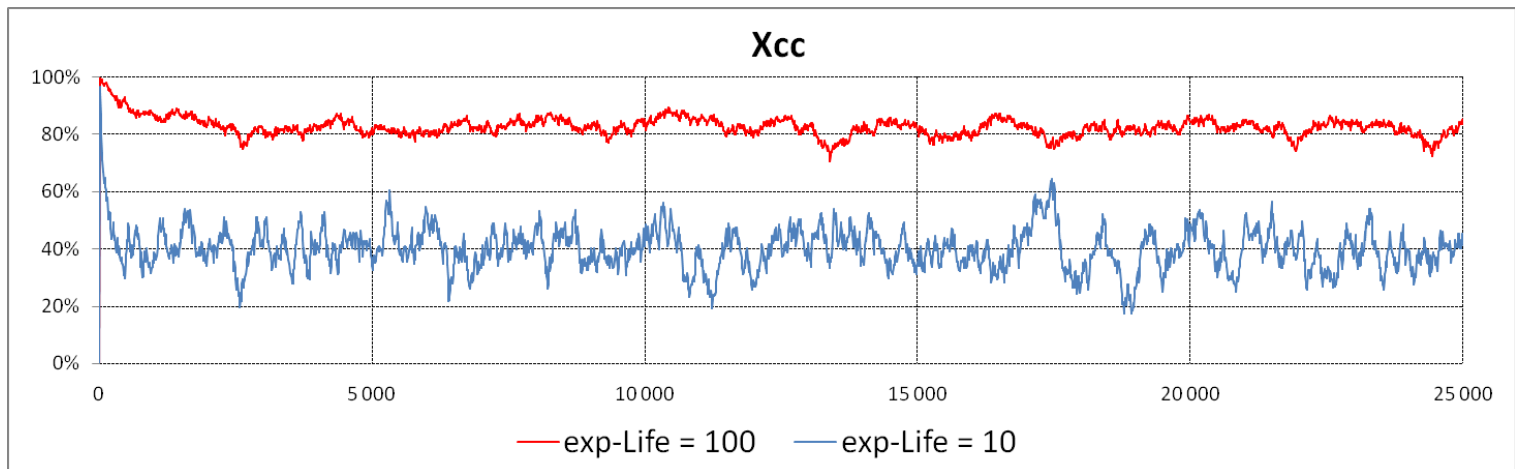


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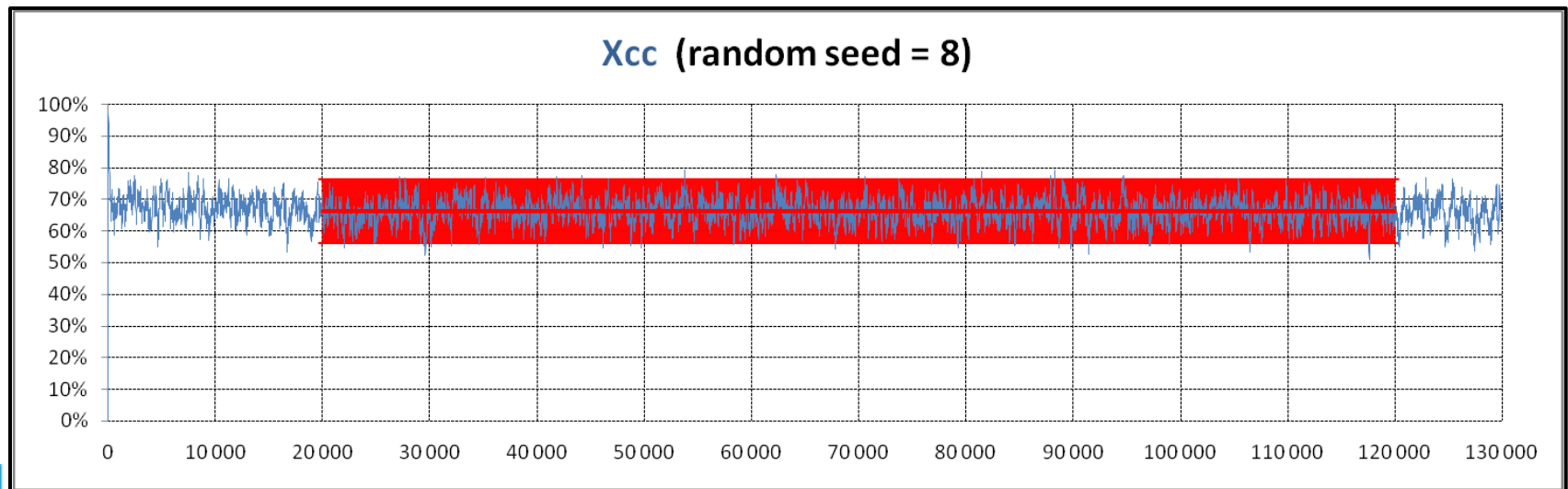


# Characterising the 2 regimes

Definition of the Cooperative regime:  $|x_{CC} - \hat{x}_{CC}^{CR}| \leq 0.1$

where  $\hat{x}_{CC}^{CR} = \bar{x}_{CC}$  in a series between periods 20 001 and period 120 000

- provided:
- $\bar{x}_{CC} \geq 0.2$
  - At least 99% samples satisfy  $|x_{CC} - \bar{x}_{CC}| \leq 0.1$



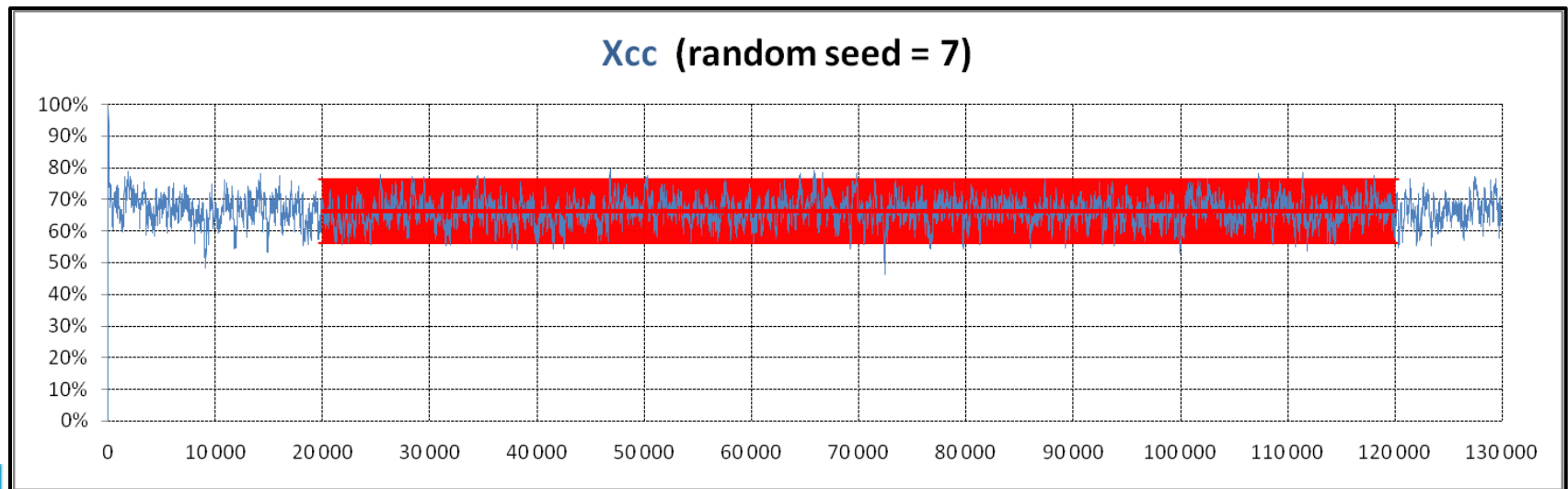
# Characterising the 2 regimes

Definition of the Cooperative regime:  $|x_{CC} - \hat{x}_{CC}^{CR}| \leq 0.1$

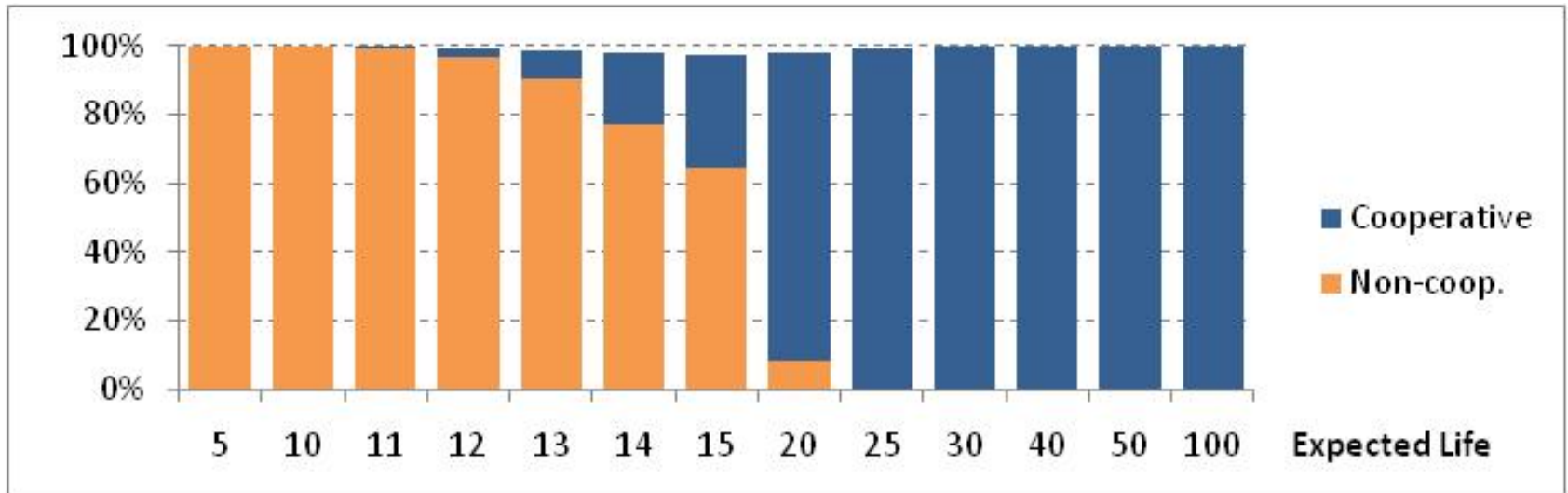
where  $\hat{x}_{CC}^{CR} = \bar{x}_{CC}$  in a series between periods 20 001 and period 120 000

provided: •  $\bar{x}_{CC} \geq 0.2$

• At least 99% samples satisfy  $|x_{CC} - \bar{x}_{CC}| \leq 0.1$  ~~X~~



# Results

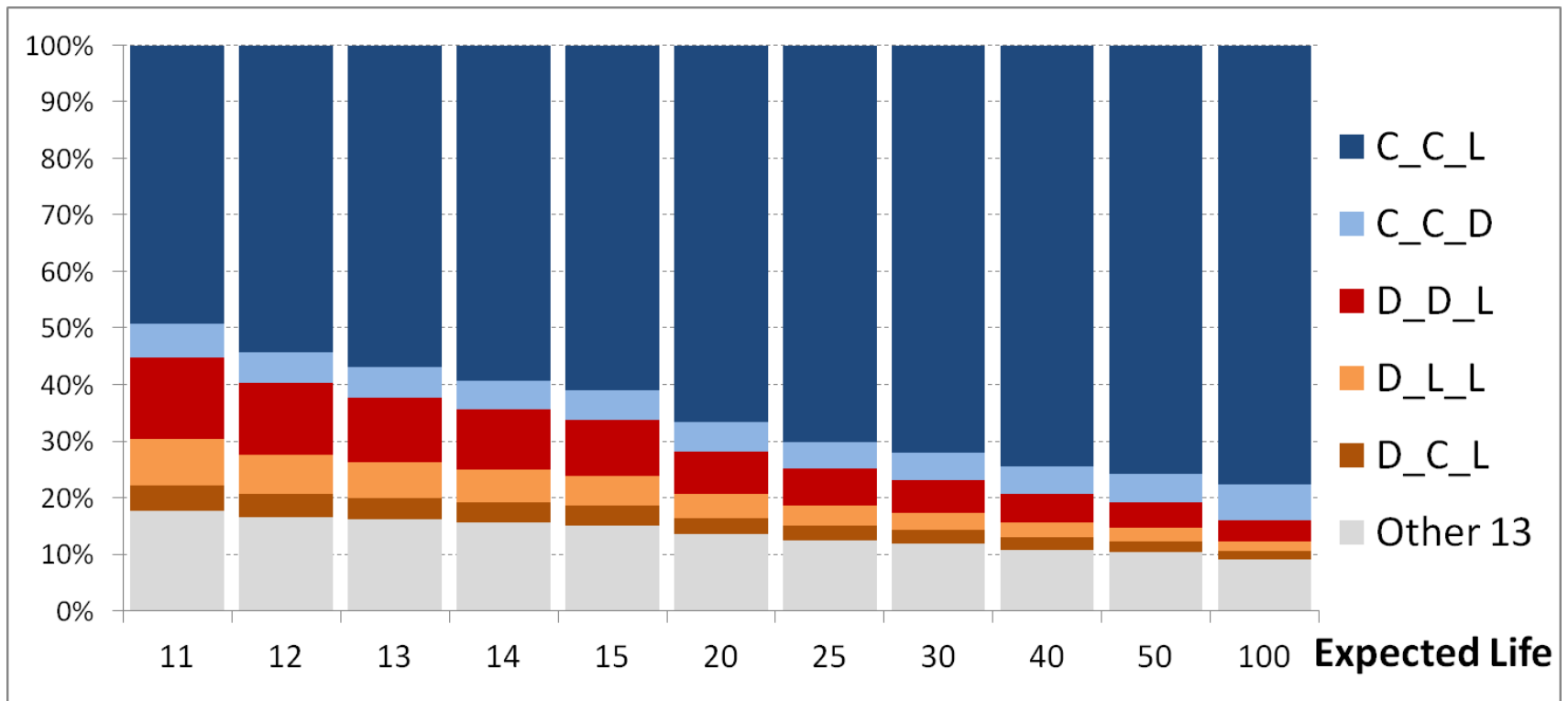


Fraction of periods spent in **the cooperative** and in **the non-cooperative** regimes as a function of the expected life *expLife*.

The values in each column are compiled over  $10^3$  simulation runs. Every run measured between periods  $3 \cdot 10^3$  and  $10^4$ , with random initial conditions. Parameterisation:  $N = 1000$ ,  $\mu = 0.05$ ,  $T = 4$ ,  $R = 3$ ,  $P = 1$ ,  $S = 0$ .



# Distribution of Strategies in the cooperative regime



# Expected Life and discount factor

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Probability that both individuals in a pair survive to the next period:

$$(1 - 1/L)^2$$

If there wasn't the option to leave, the effective discount factor would be:

$$\delta = (1 - 1/L)^2$$



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# Mean-field approximation

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## Time-homogeneous Markov chain

Ergodic if  $\expLife < \infty$  and  $\mu > 0$ .

## Mean-field approximation

Time-homogeneous system of ordinary differential equations with hundreds of state variables.

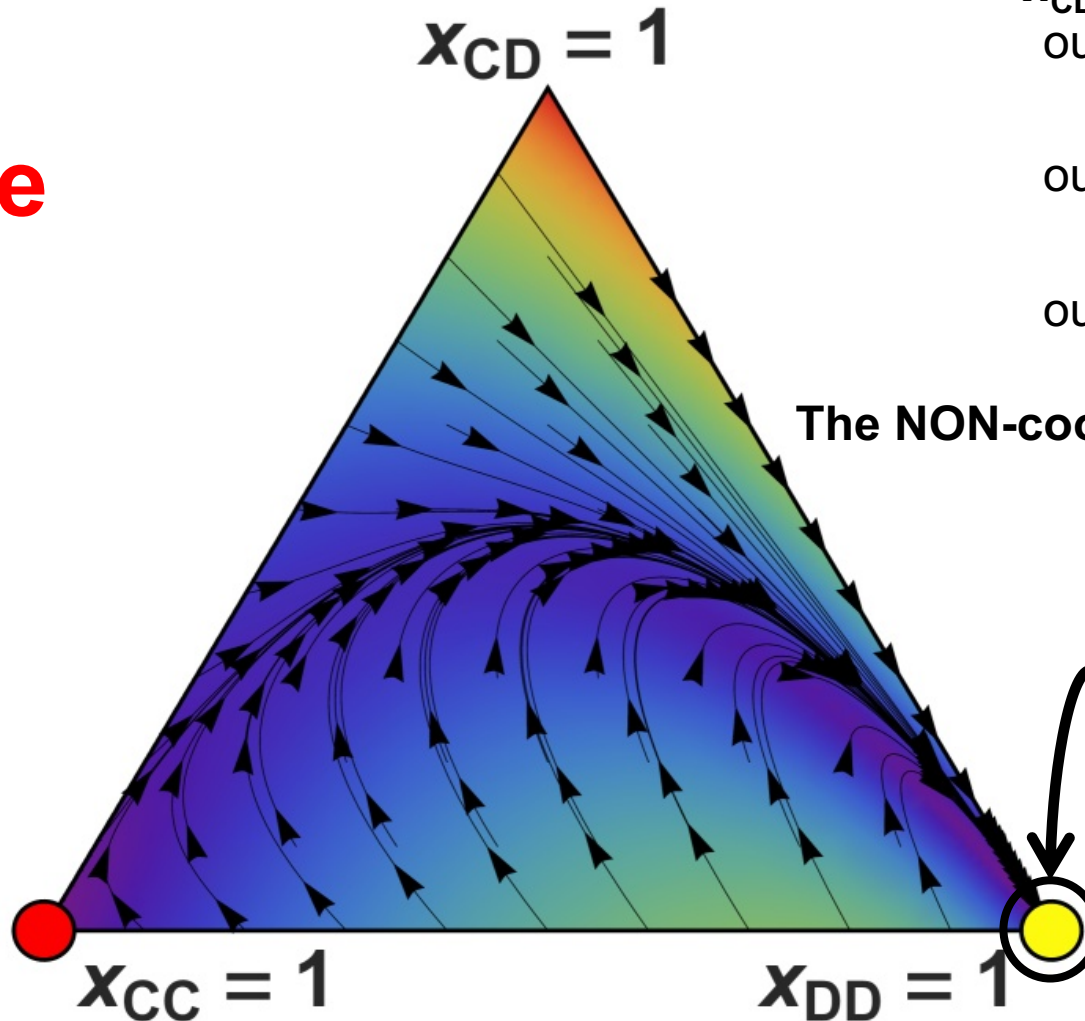
## Reduced mean-field approximation to calculate the level of cooperation in the cooperative regime

Focus only on the strategies that the numerical simulations single out as the most prevalent in the cooperative regime: **C\_C\_L** and family **D\_X\_L**.



# Mean-field approximation

**Short life**  
 $expLife = 1$



$x_{CD}$  : share of CD, DC outcomes in a period

$x_{CC}$  : share of CC outcomes in a period

$x_{DD}$  : share of DD outcomes in a period



# Mean-field approximation

**Moderate life**  
*expLife* = 10

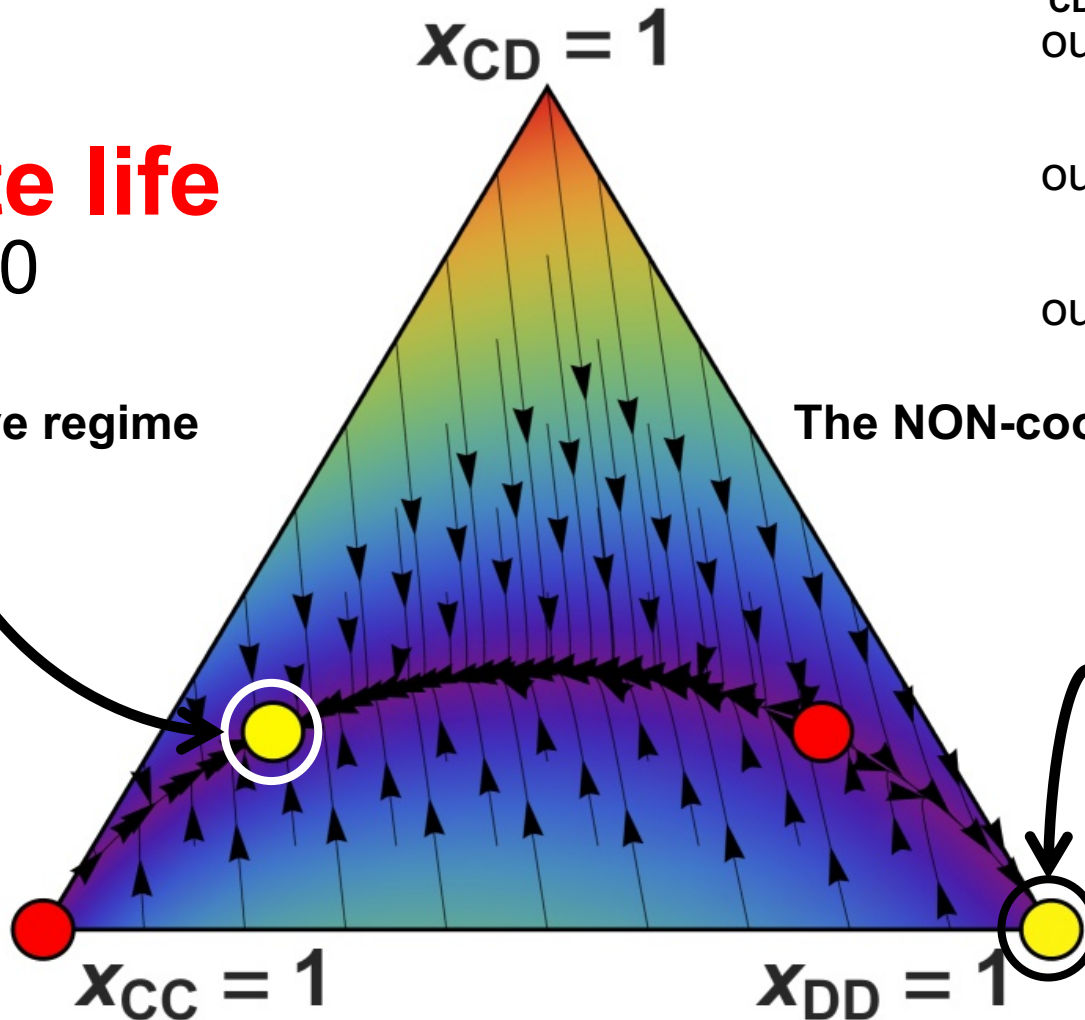
$x_{CD}$  : share of CD, DC  
outcomes in a period

$x_{CC}$  : share of CC  
outcomes in a period

$x_{DD}$  : share of DD  
outcomes in a period

The cooperative regime

The NON-cooperative regime

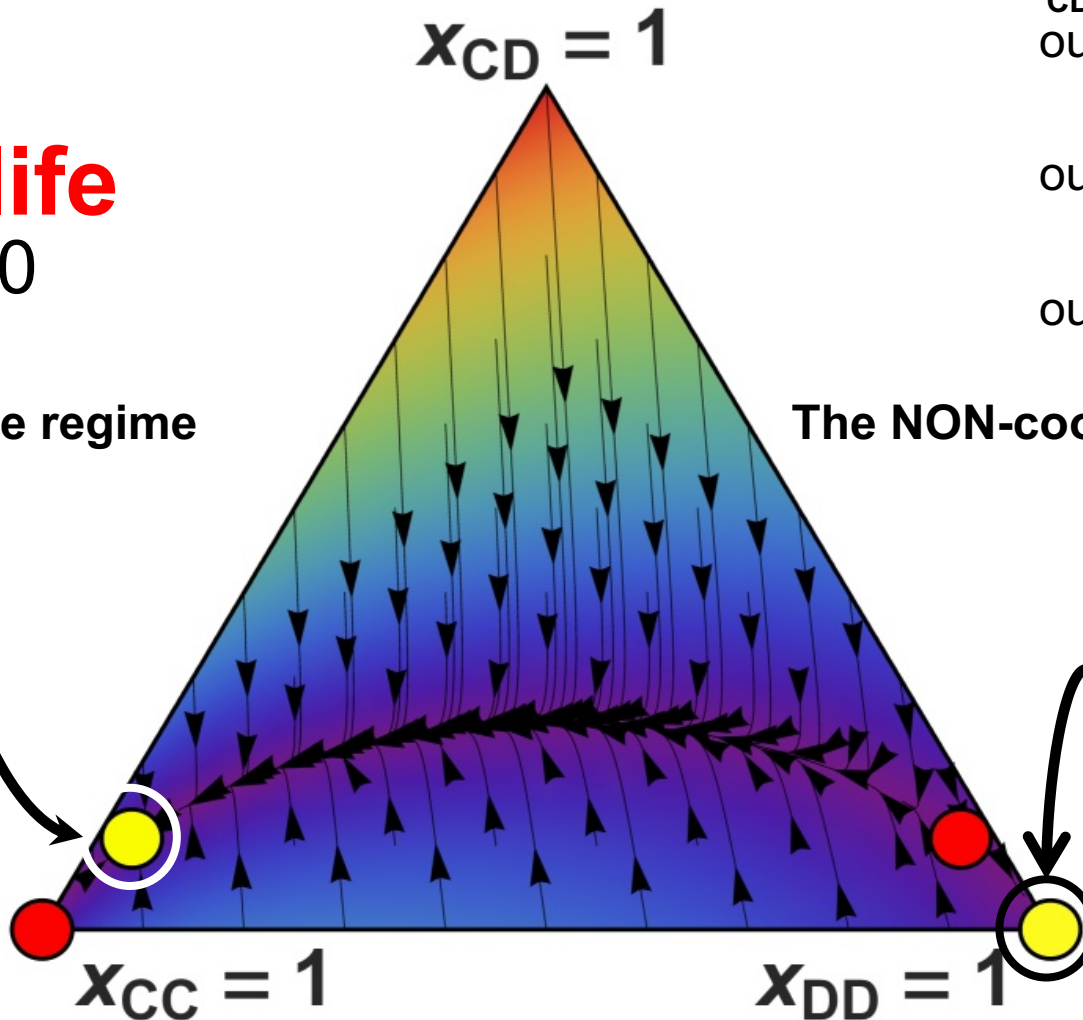


# Mean-field approximation

**Longer life**  
*expLife* = 20

The cooperative regime

The NON-cooperative regime



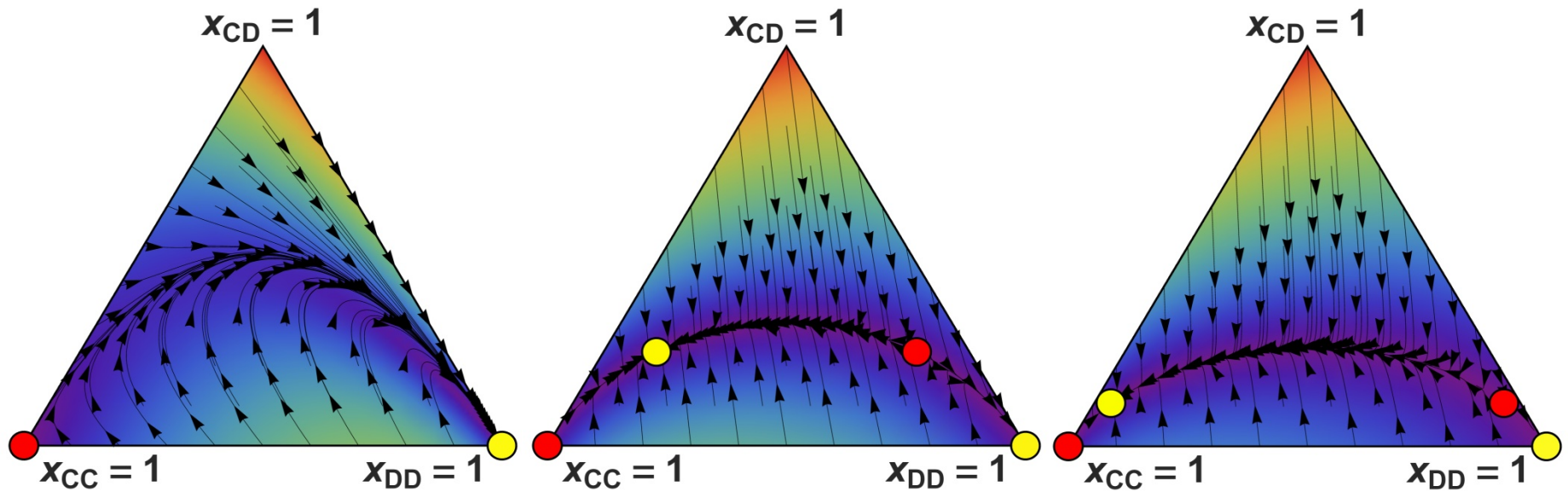
$x_{CD}$  : share of CD, DC outcomes in a period

$x_{CC}$  : share of CC outcomes in a period

$x_{DD}$  : share of DD outcomes in a period



# Mean-field approximation



**Short Life**

$expLife = 1$

**Moderate Life**

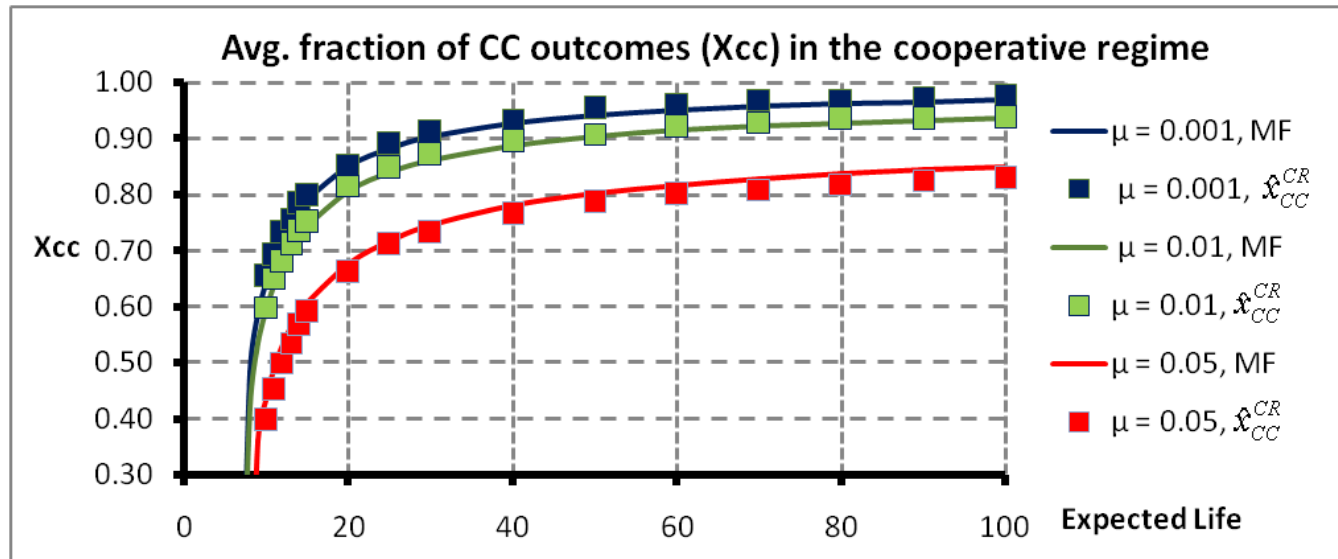
$expLife = 10$

**Longer Life**

$expLife = 20$



# Mean-field approximation



Average values of the **level of cooperation** ( $x_{CC}$ ) in the cooperative regime as a function of the individuals' expected life  $expLife$ , both in the mean-field approximation and in the stochastic simulations with different values of  $\mu$ .

Parameterisation:  $N = 1000$ ,  $T = 4$ ,  $R = 3$ ,  $P = 1$ ,  $S = 0$ .



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# Conclusions

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- A simple mechanism of conditional dissociation can explain the evolutionary emergence of cooperation.
- Key factor: expected lifespan of the individuals.  
It is sufficient that lifespans are only moderately long.
- Cooperative regime mostly composed of “*conditional dissociators*”, even in the presence of more popular strategies like Tit-For-Tat.





# Should I stay or should I go?

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the Evolutionary Emergence of Cooperation

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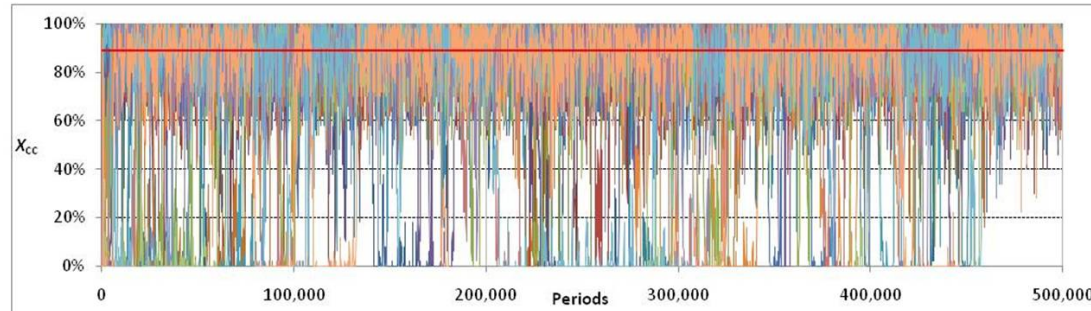
University of Valladolid (Spain)

University of Burgos (Spain)

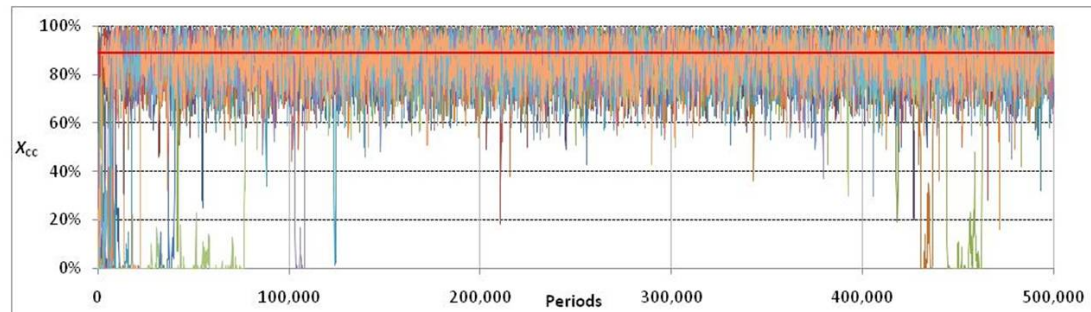
European University Institute (Italy)

# Different populations

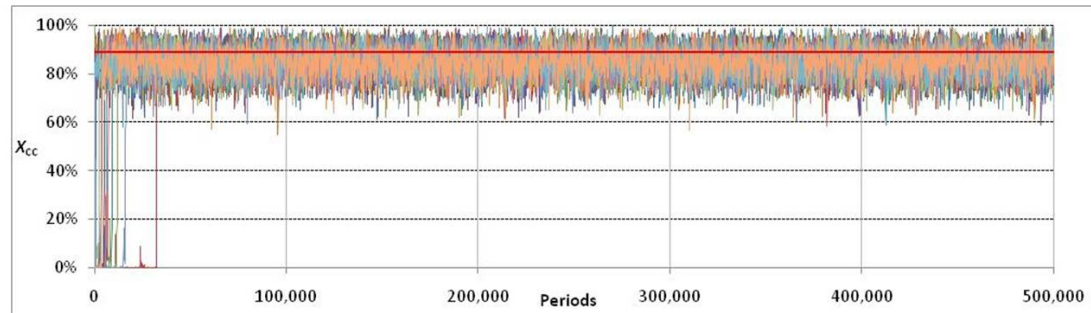
$N = 100$



$N = 200$



$N = 400$



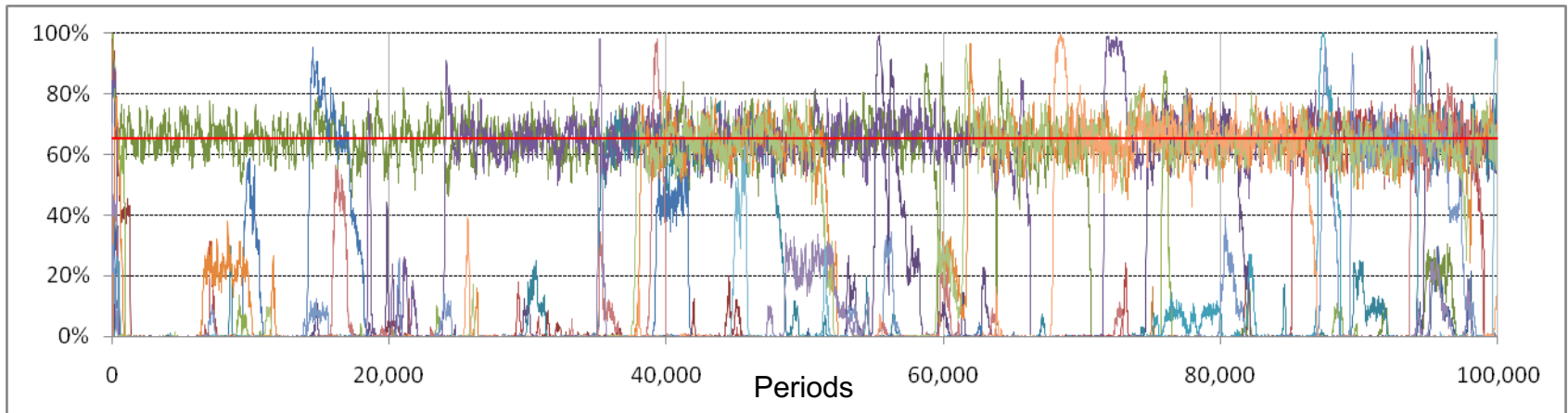
$expLife = 25, \mu = 0.01, T = 4, R = 3, P = 1, S = 0.$

The horizontal line at  $x_{cc} = 89\%$  shows the stable degree of cooperation in the mean-field approximation



# Regime changes

CC outcomes



$N = 1000$ ,  $expLife = 10$ ,  $\mu = 0.001$ ,  $T = 4$ ,  $R = 3$ ,  $P = 1$ ,  $S = 0$ .

The horizontal line at  $x_{CC} = 65.5\%$  shows the stable degree of cooperation in the mean-field approximation

