Leaving undesirable partners

- A sufficient condition to explain the evolutionary emergence of cooperation

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Paper: http://dx.doi.org/10.1016/j.jedc.2014.06.007
Online model: http://luis.izqui.org/models/LeLeLe/index.html
5-minute video: http://luis.izqui.org/papers/Izquierdo_Izquierdo_Vega-Redondo_2014.html

Outline

- Introduction
- The question and the approach
- The model
- Simulation results
- A mean-dynamics approximation
- A closed-form solution
- Conclusions

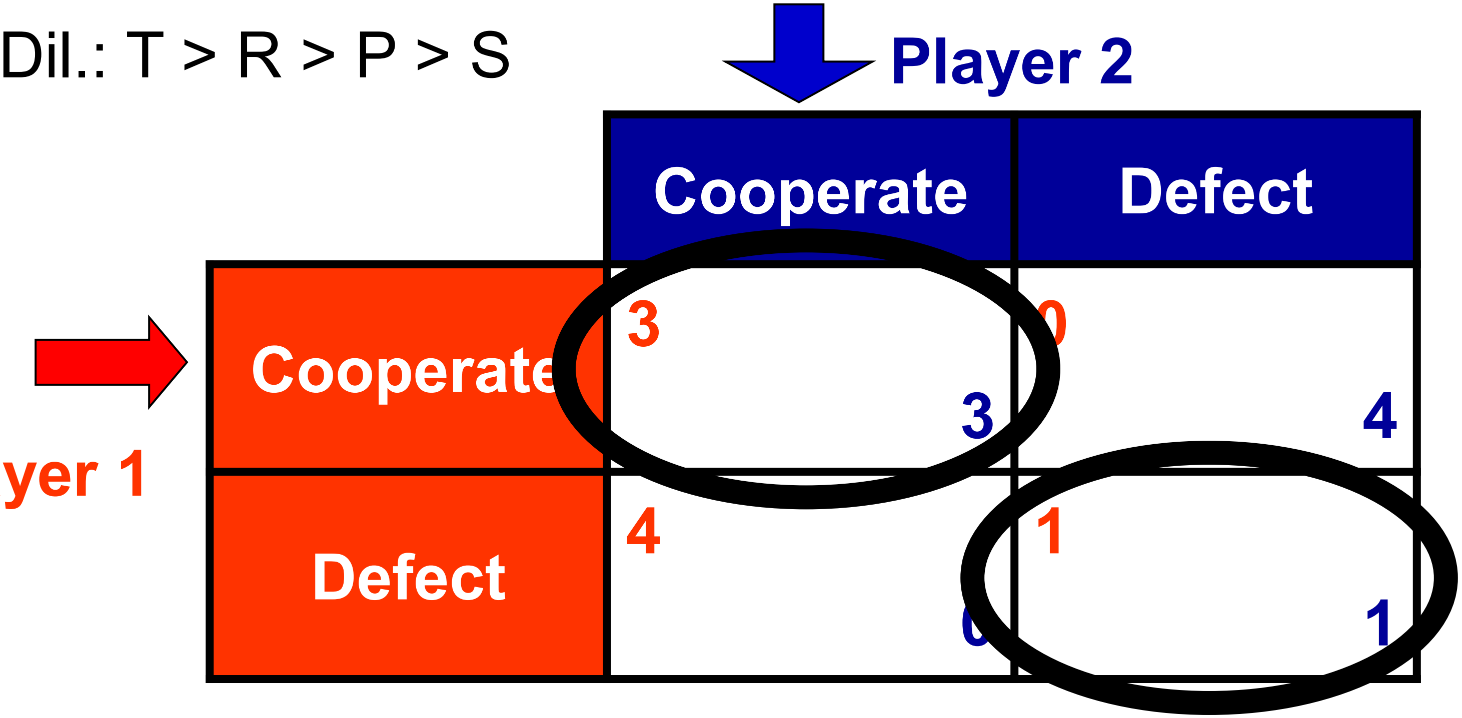
Introduction

How did cooperative behaviour evolve?

Social Dilemmas

The Prisoner's Dilemma

Pris. Dil.: T > R > P > S



Player 2

		Cooperate	Defect
Player 1	**Cooperate**	3 3	0 4
	Defect	4 0	1 1

Social Dilemmas

The Prisoner's Dilemma

Cooperation is mutually beneficial, but very fragile.

Pris. Dil.: T > R > P > S

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3 3	0 4
	Defect	4 0	1 1

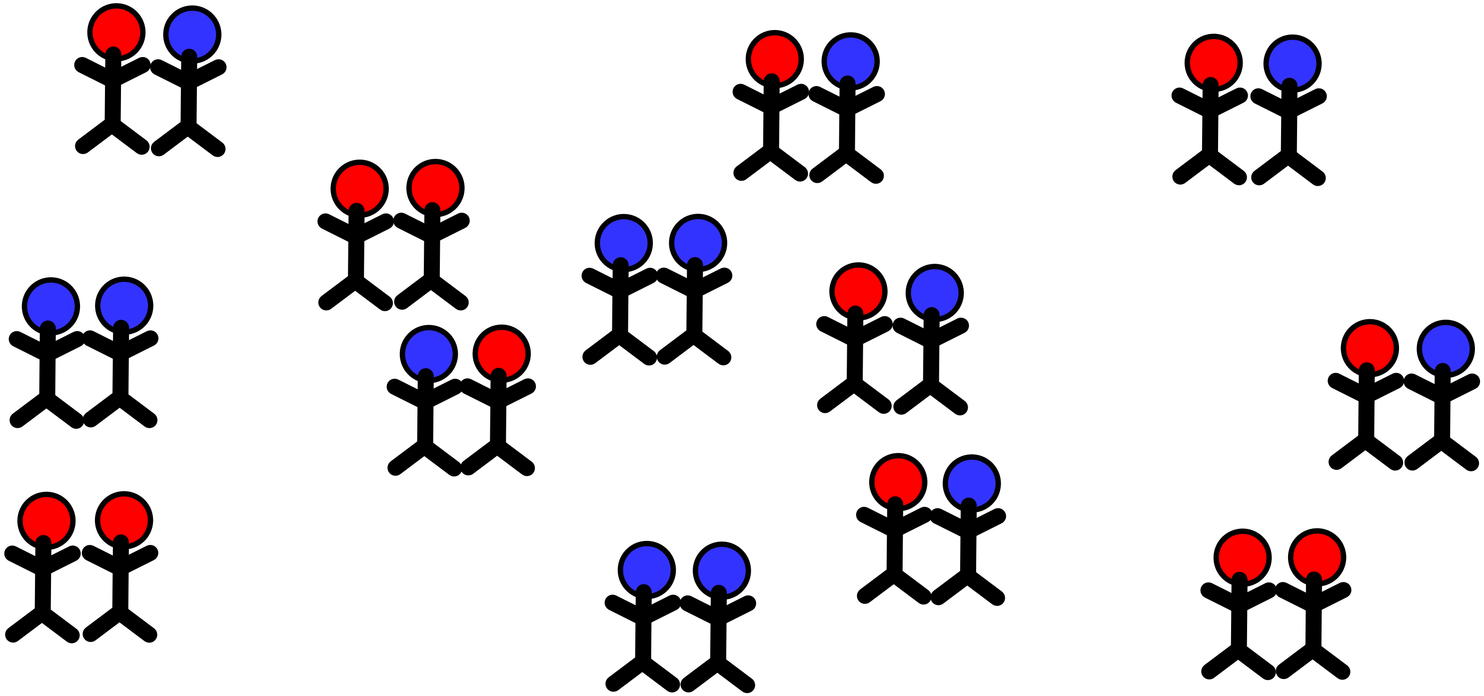
Positive assortment

Processes or conditions that allow cooperators to preferentially interact among themselves.

Expected Payoff:

Pris. Dil.: T > R > P > S

x_C : probability of interacting with a cooperator
x_D : probability of interacting with a defector
(1 - x_C)



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Pris. Dil.: T > R > P > S

$$\begin{array}{lcccccc} \text{For Cooperators:} & \boxed{R} & x_C & + & \boxed{S} & (1 - x_C) \\ & \wedge & & & \wedge & \\ \text{For Defectors:} & \boxed{T} & x_C & + & \boxed{P} & (1 - x_C) \end{array}$$

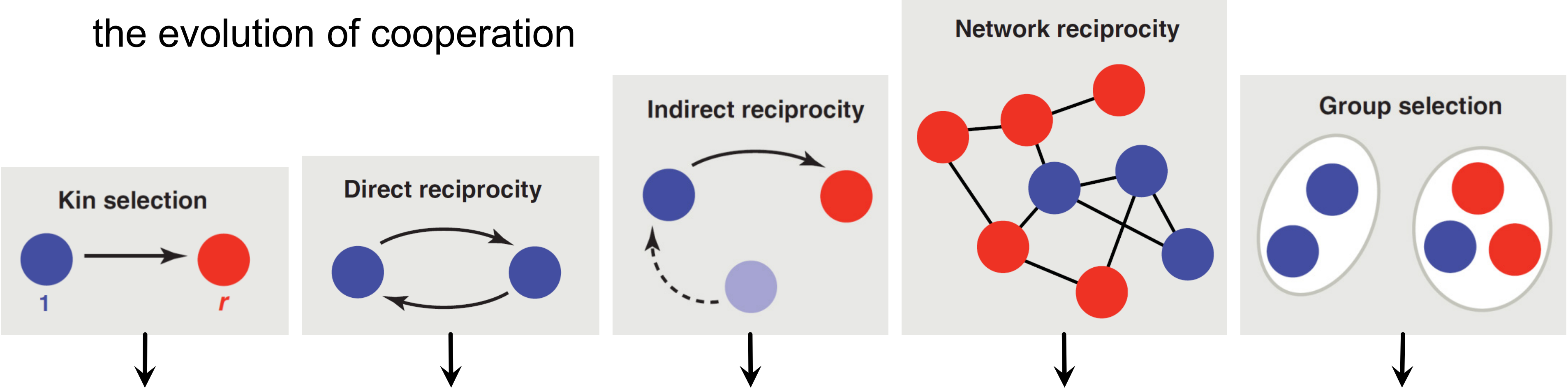
$$\begin{array}{lcccccc} \text{For Cooperators:} & \boxed{R} & x_{C|C} & + & \boxed{S} & (1 - x_{C|C}) \\ & \wedge & \color{red}{\vee} & & \wedge & \color{red}{\wedge} \\ \text{For Defectors:} & \boxed{T} & x_{C|D} & + & \boxed{P} & (1 - x_{C|D}) \end{array}$$

The probability of interacting with a cooperator must be greater for a cooperator than for a defector

Introduction

How did cooperative behaviour evolve?

Five mechanisms for
the evolution of cooperation



Requires genetic relatedness

Requires recognition and memory

Requires recognition and social monitoring (reputation)

Imposes exogenous structure of interaction

Imposes exogenous structure of interaction and multilevel selection

🟦 Cooperators 🟥 Defectors

Nowak, M. A. (2006) Five rules for the evolution of cooperation. *Science* 314, 1560-1563.

Introduction

Interested in conditional assortment

Situations where individuals can influence with whom they interact (or not), or at least for how long they do it.

- *Pre-interaction* partner selection (or refusal)
  - Requires advanced cognitive abilities.
  - E.g.: Indirect reciprocity (reputation), external signals or tags...
- *Post-interaction* partner refusal (**conditional dissociation**)
  - It does not require memory or the ability to anticipate the behaviour of new partners.
  - It only requires the capacity to escape an unpleasant situation.

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The question and the approach

- Can a simple mechanism of **conditional dissociation** explain, **only by itself**, the evolutionary emergence and stability of cooperation among unrelated individuals?
- **The simplest model we can think of,** to study the effect of conditional dissociation isolated from other mechanisms.
- **Unbiased selection of strategies**
  All individuals in the population share a common capability to gather information and to condition their actions on that information, and then allow for *every* possible strategy within such a uniform setup.
- **Beyond the identification of "winning" strategies**
  Also concerned with the issue of whether some well-defined pattern of aggregate behaviour eventually emerges.
  (Even under perpetual turnover in strategy choice.)

The approach

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	**3** **3**	**0** **4**
	Defect	**4** **0**	**1** **1**

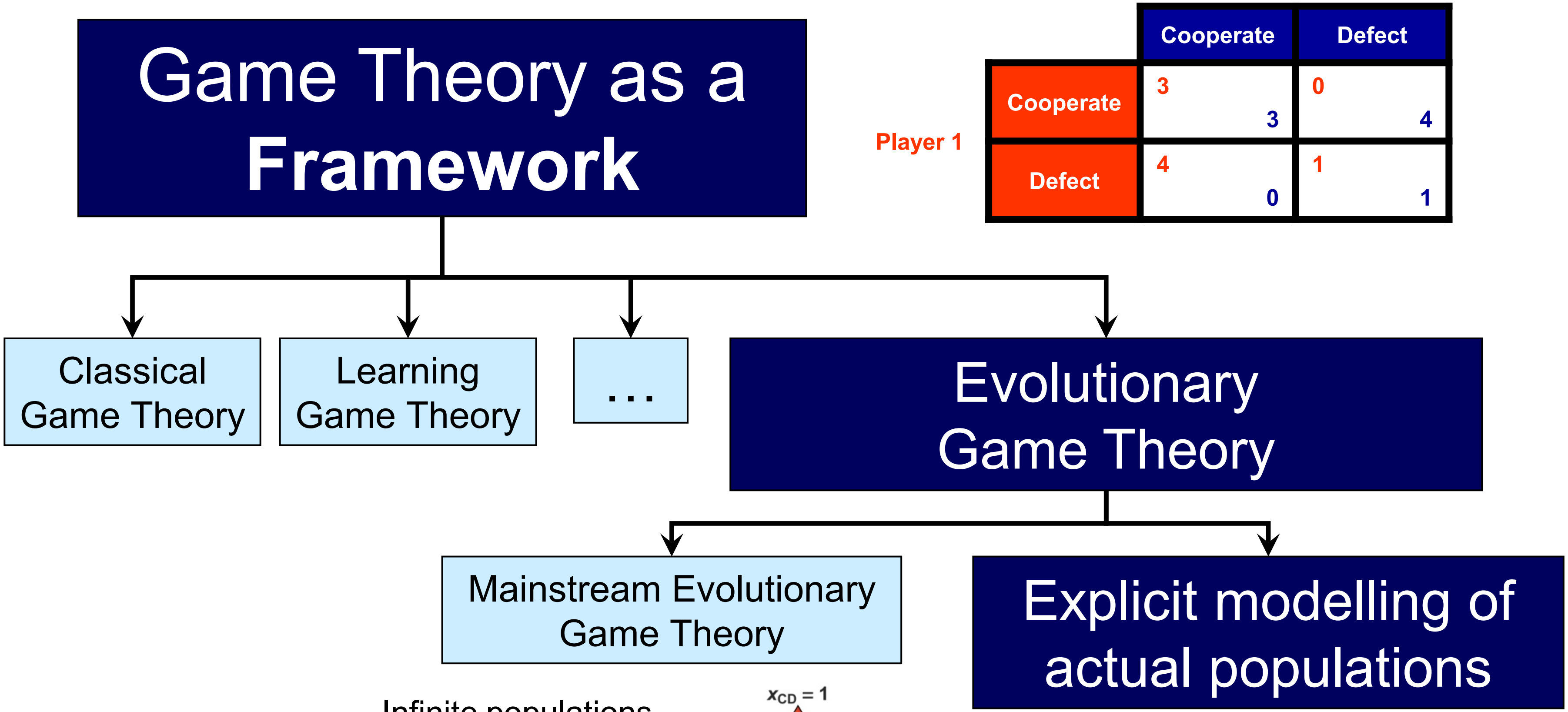
Game Theory as a **Framework**

- Classical Game Theory
- Learning Game Theory
- …
- Evolutionary Game Theory

- **Dynamic process, where entities which are *more successful* at a particular time will have the best chance of being present in the future.**

The approach

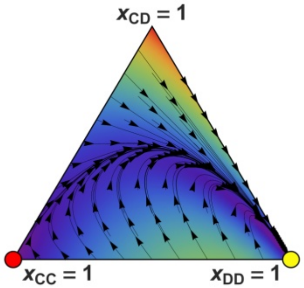
		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3 3	0 4
	Defect	4 0	1 1

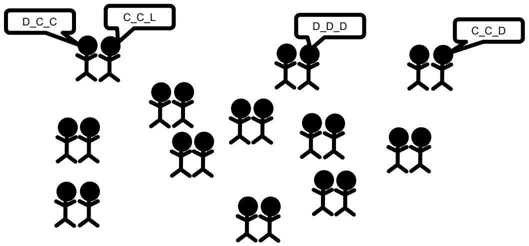


Game Theory as a Framework

- Classical Game Theory
- Learning Game Theory
- …
- Evolutionary Game Theory
  - Mainstream Evolutionary Game Theory
  - Explicit modelling of actual populations

Infinite populations
Random Pairings
Proportional Fitness Rule
Arbitrarily small mutations
No random drift





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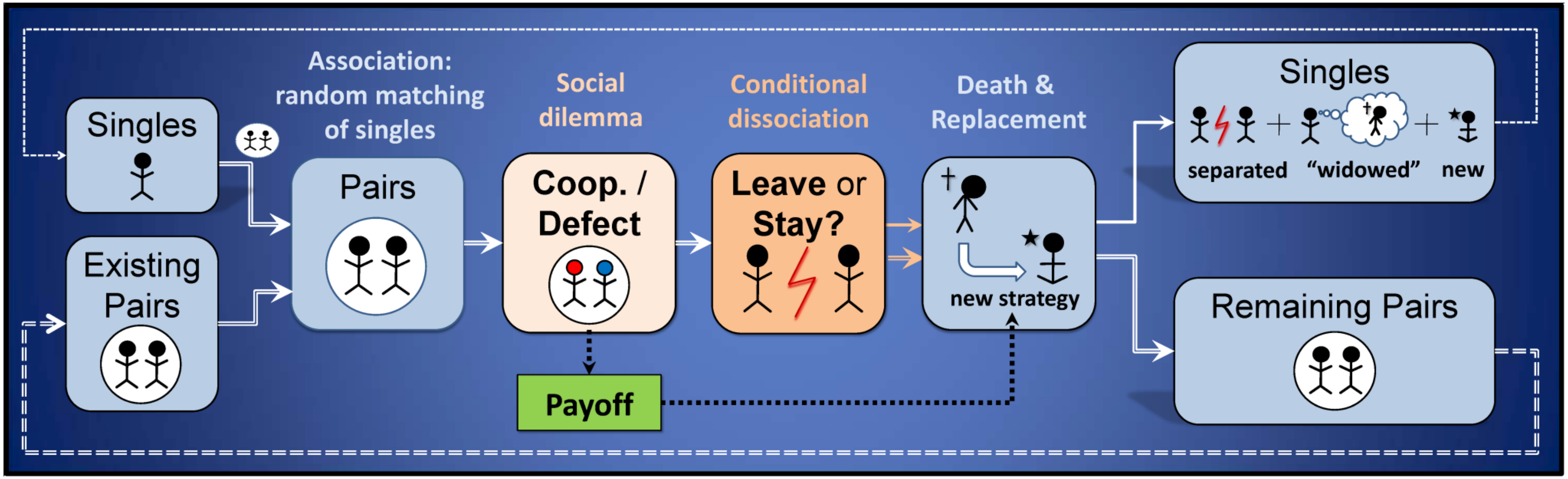
The game (Prisoner’s Dilemma)

The Prisoner’s Dilemma: Payoffs

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3 3	0 4
	Defect	4 0	1 1

The timeline



Sequence of events within each time period

The strategies: X1_X2_X3

X1

{C, D}

Action in the Prisoner's Dilemma

C: Cooperate

D: Defect

X2

{S, L}

Action to take if one's current partner cooperated

S: Stay with partner.

L : Leave the partner (and go to the pool of singles).

X3

{S, L}

Action to take if one's current partner defected

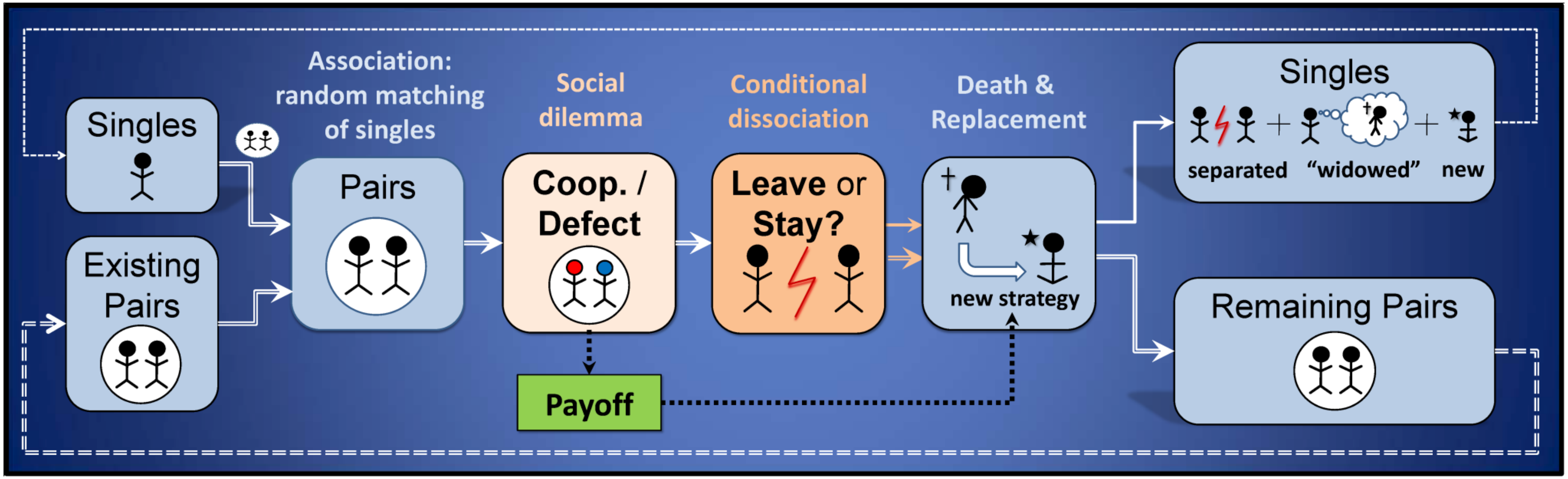
S: Stay with partner.

L : Leave the partner (and go to the pool of singles).

Cooperate and leave after a partner's defection: C_S_L

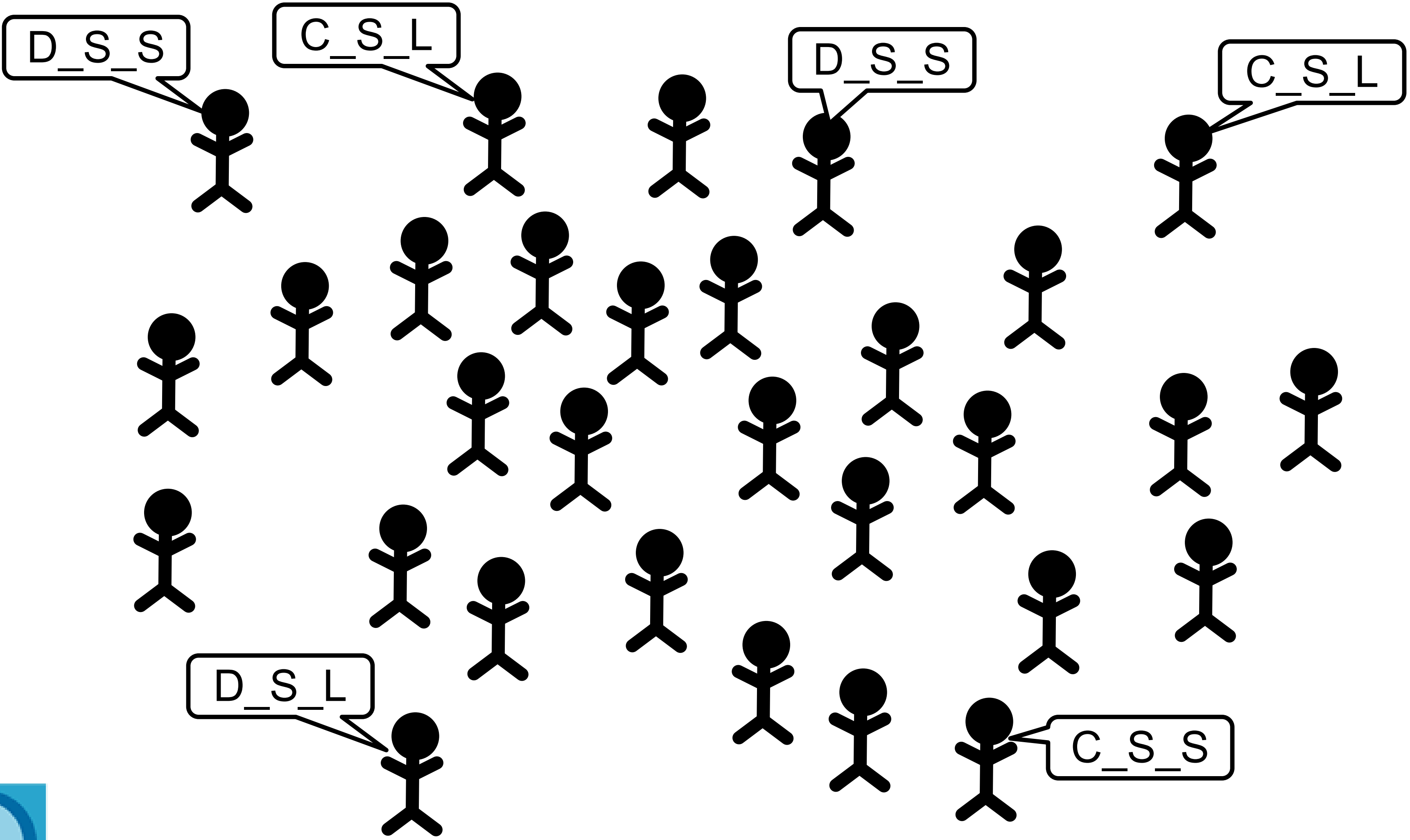
(Conditional dissociator or Out-for-Tat)

The timeline

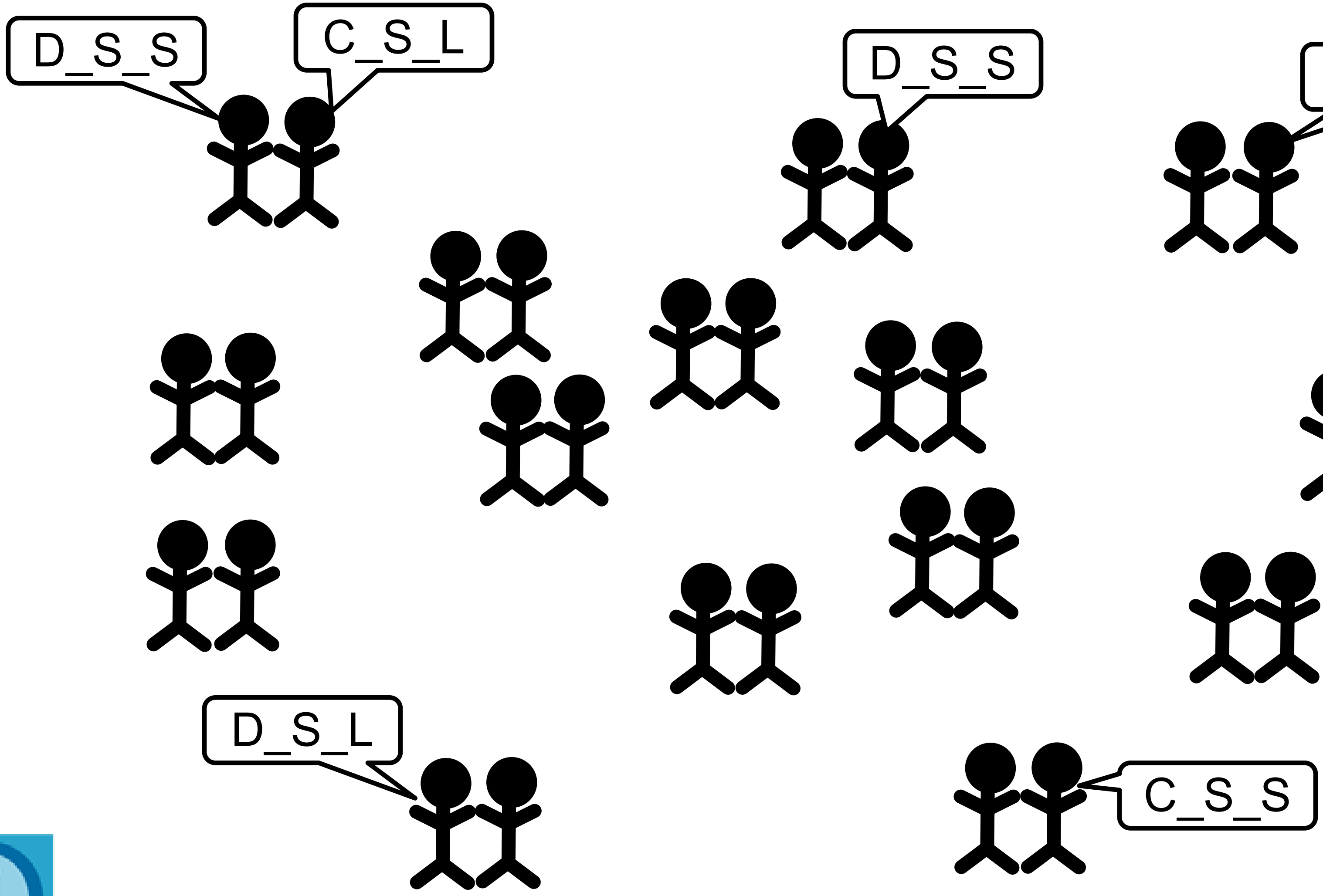


Sequence of events within each time period

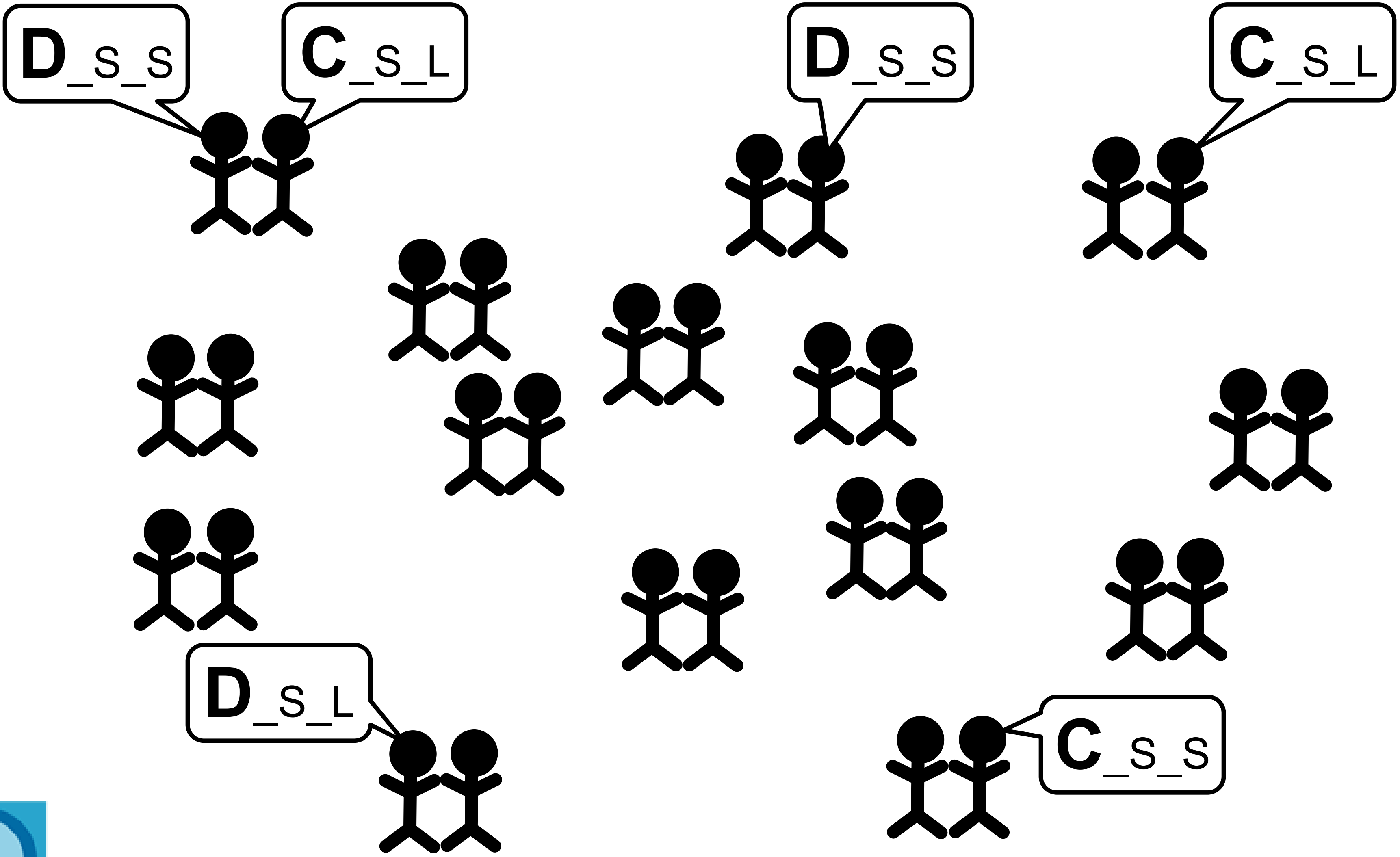
The initial population



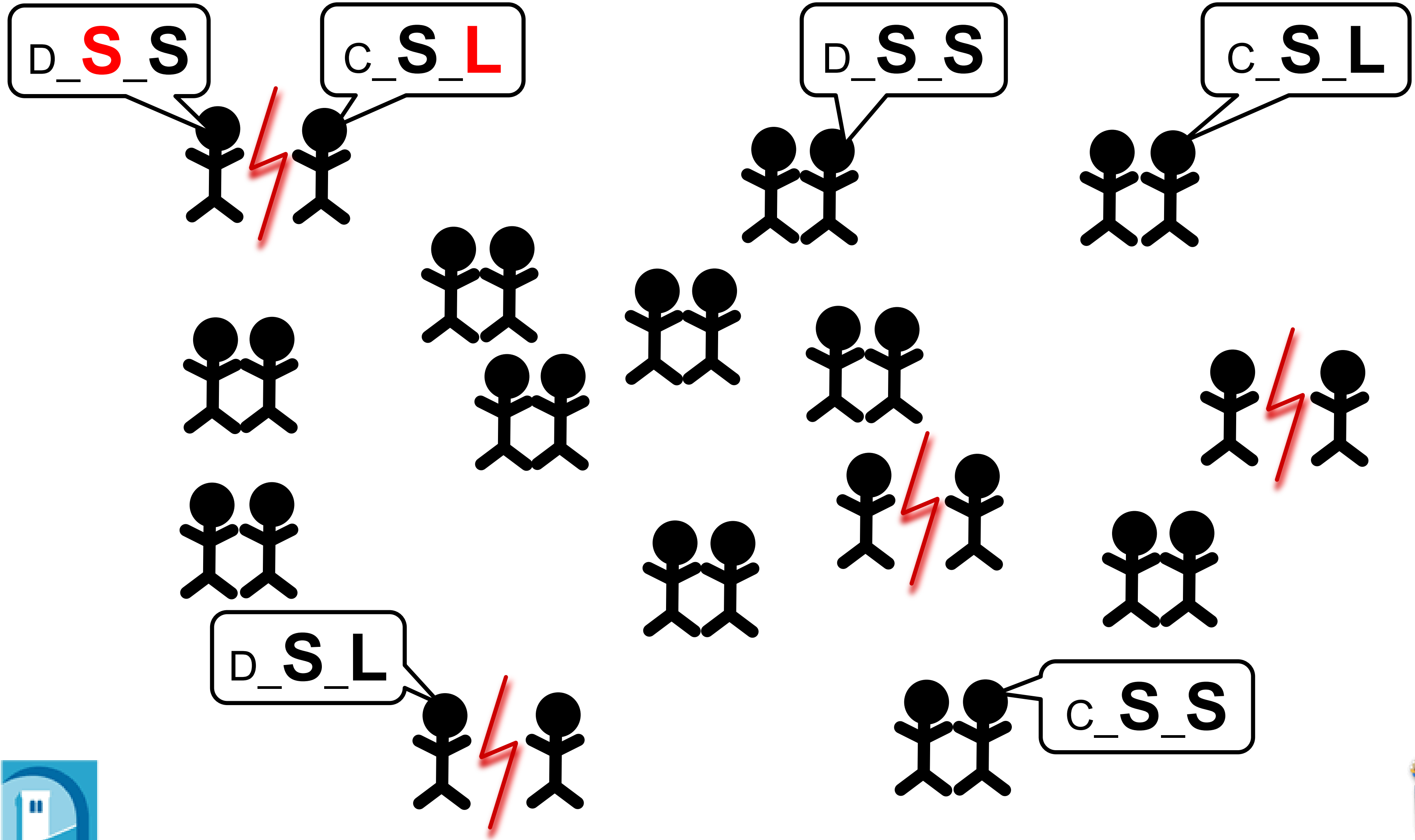
The random pairing of singles



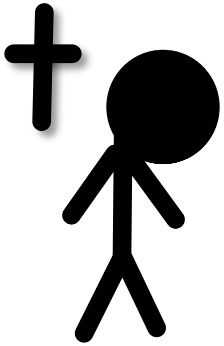
The Prisoner's Dilemma game



Should I stay or should I go?



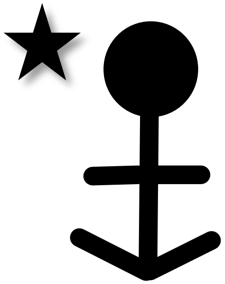
Death and Replacement



Individuals randomly die, an event that occurs independently for each individual with probability *p* in each time period.

Hence, **the lifespan of an individual is geometrically distributed with mean-value** ***expLife*** = 1/*p*.

Each dead individual is immediately replaced by a new entrant



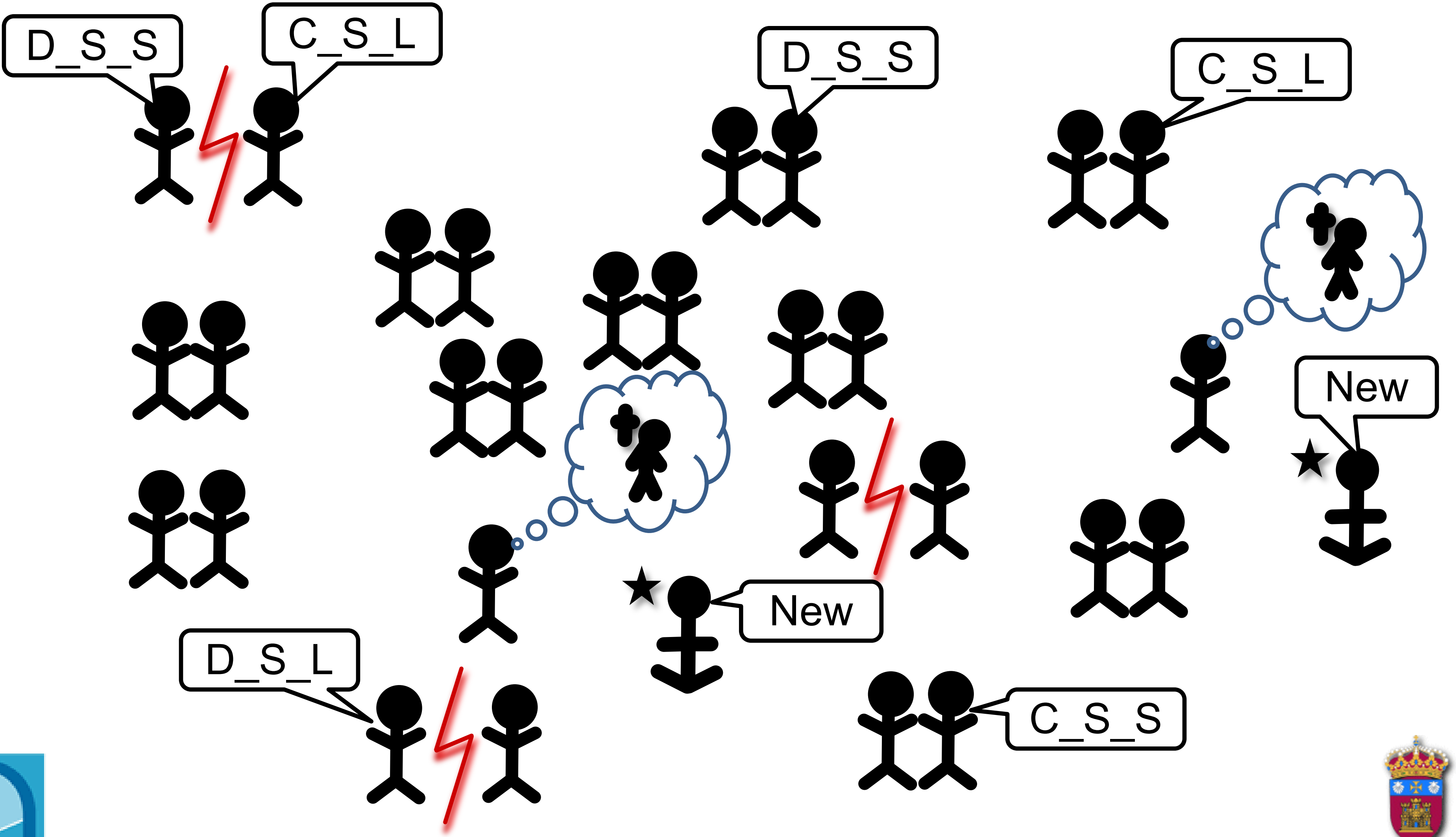
new strategy

The probability of adopting a certain strategy is proportional to its aggregate payoff.

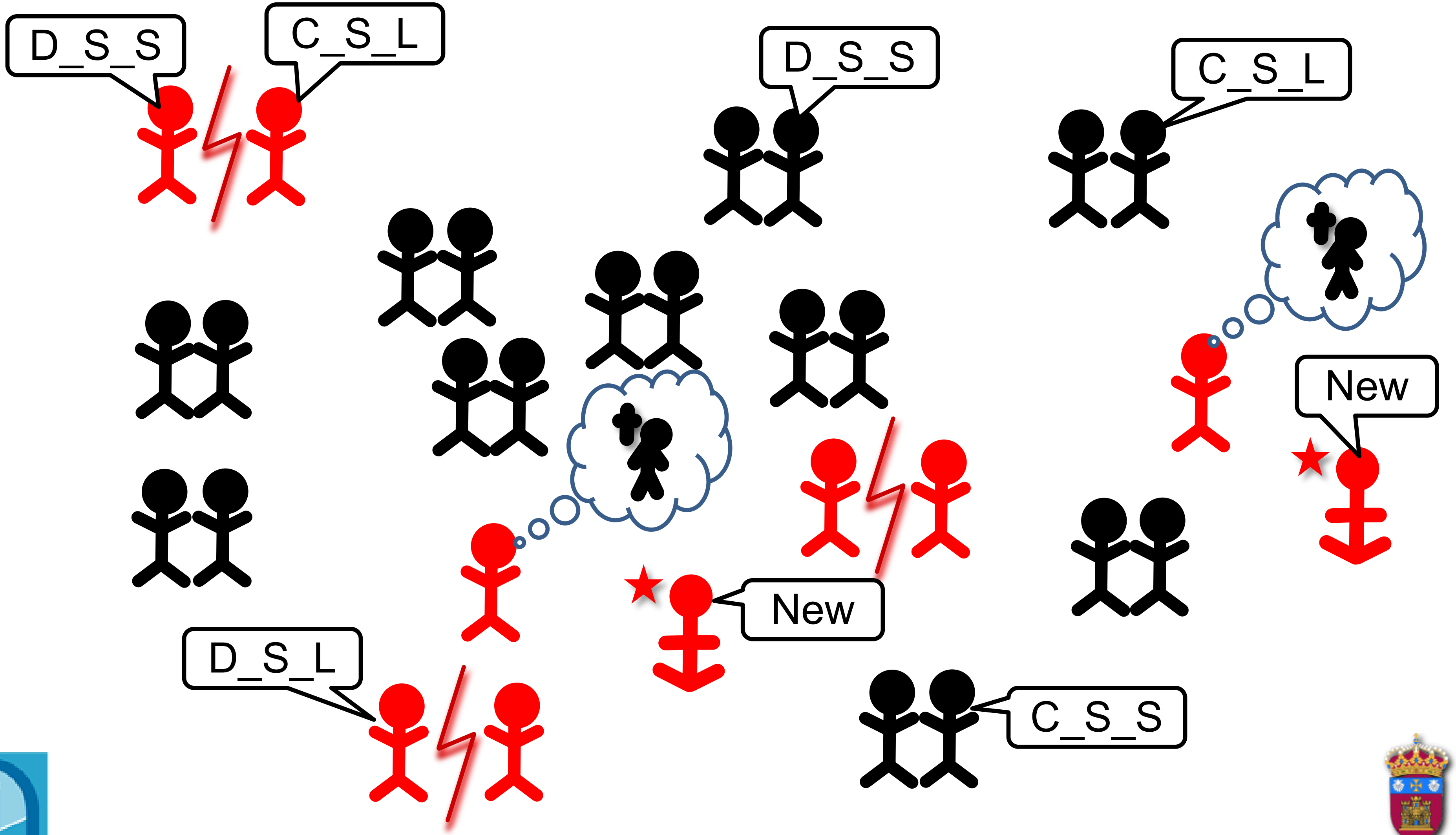
(Reproduction is proportional to payoffs and population size is kept constant.)

Random mutations occur with probability μ.

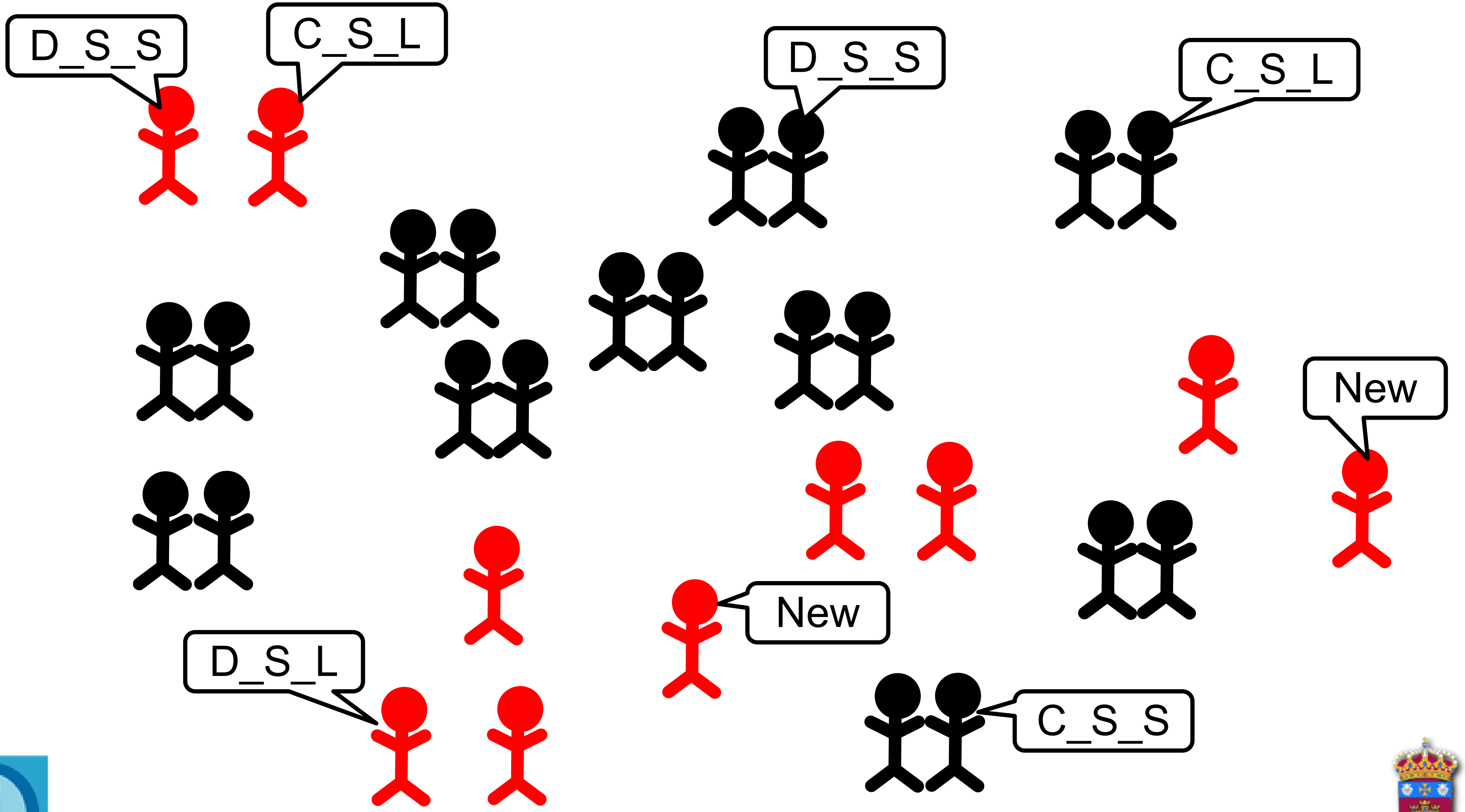
The population for the next step



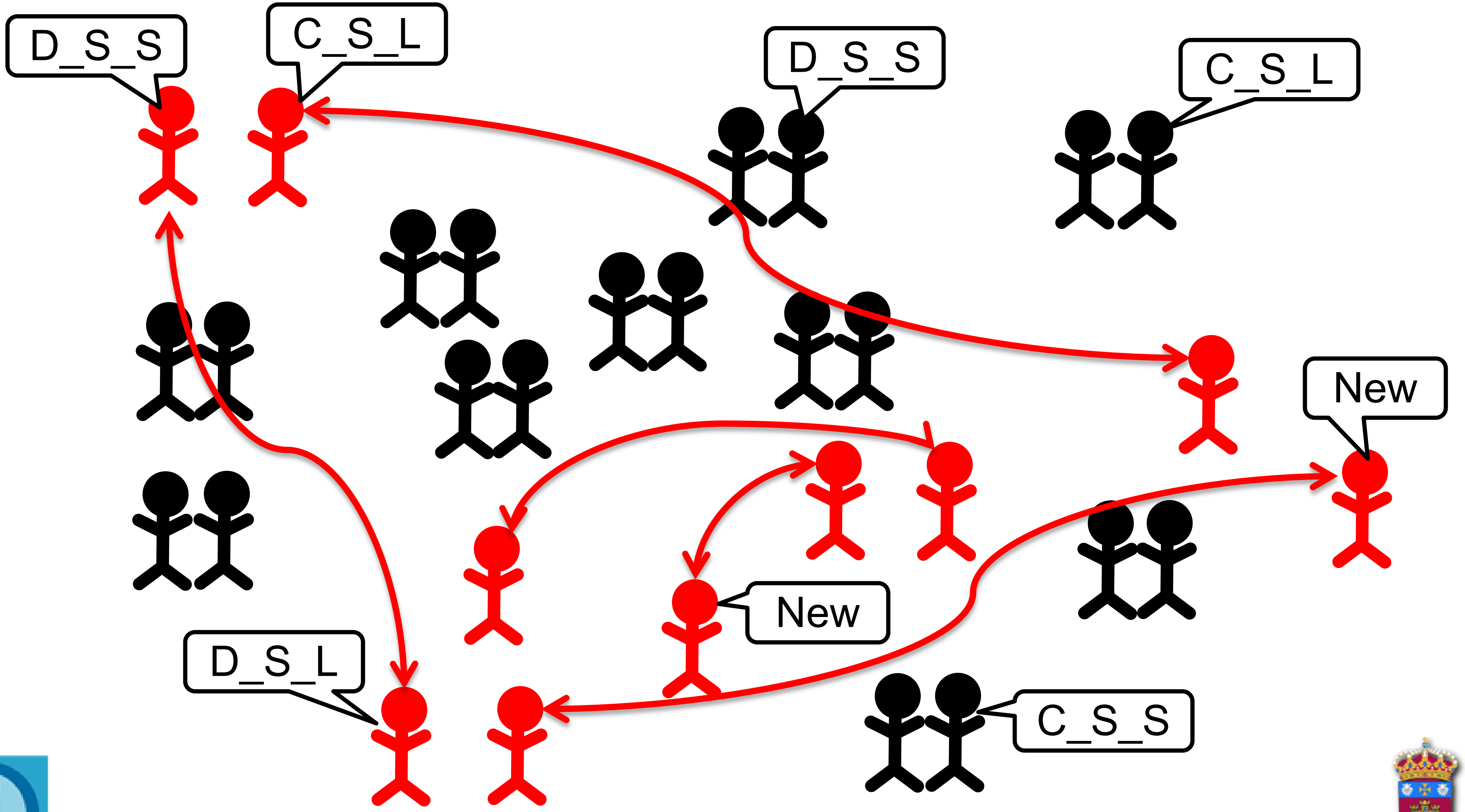
The population for the next step



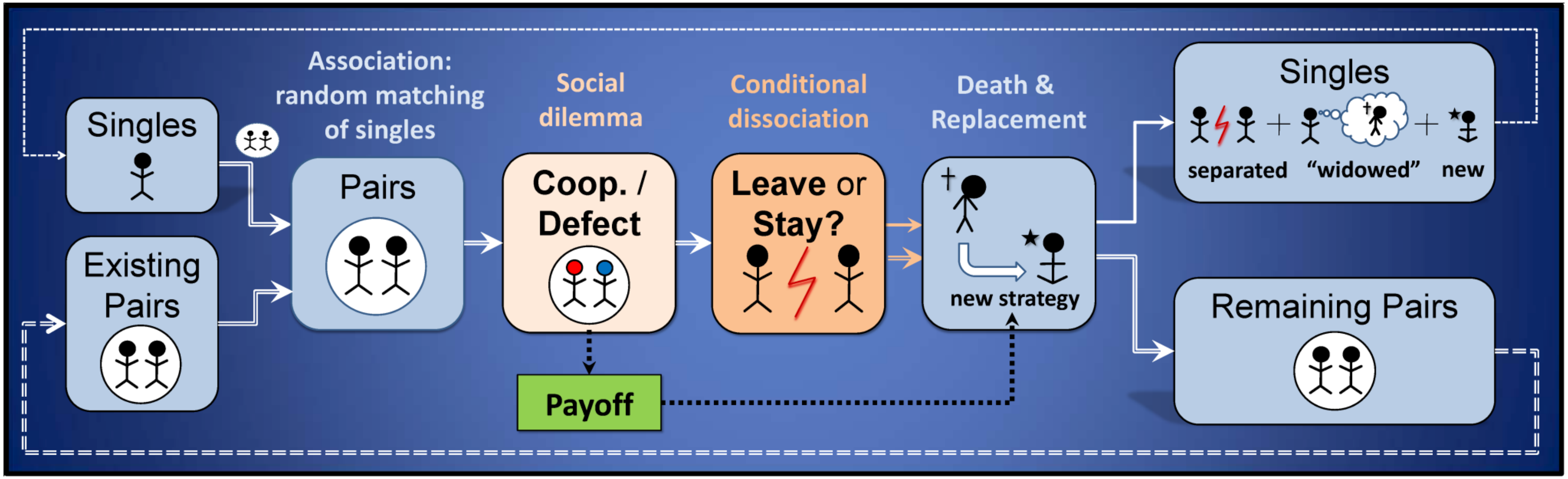
The random pairing of singles



The random pairing of singles

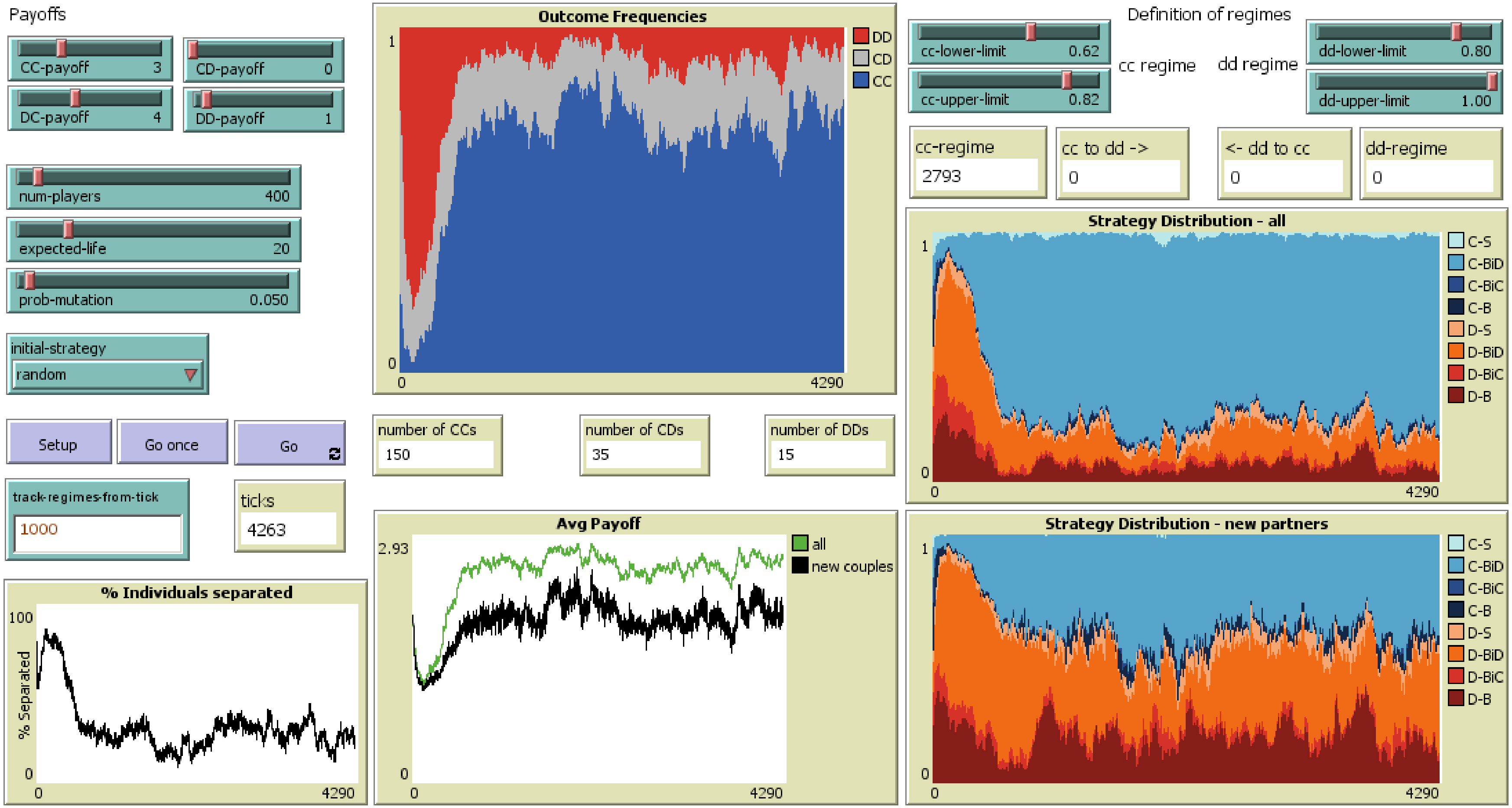


The timeline



Sequence of events within each time period

The implementation of the model



CC-payoff 3

CD-payoff 0

DC-payoff 4

DD-payoff 1

num-players 400

expected-life 20

prob-mutation 0.050

initial-strategy: random

track-regimes-from-tick: 1000

ticks: 4263

cc-lower-limit 0.62

cc-upper-limit 0.82

dd-lower-limit 0.80

dd-upper-limit 1.00

cc-regime: 2793

cc to dd ->: 0

<- dd to cc: 0

dd-regime: 0

number of CCs: 150

number of CDs: 35

number of DDs: 15

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Three scenarios

Key factor:

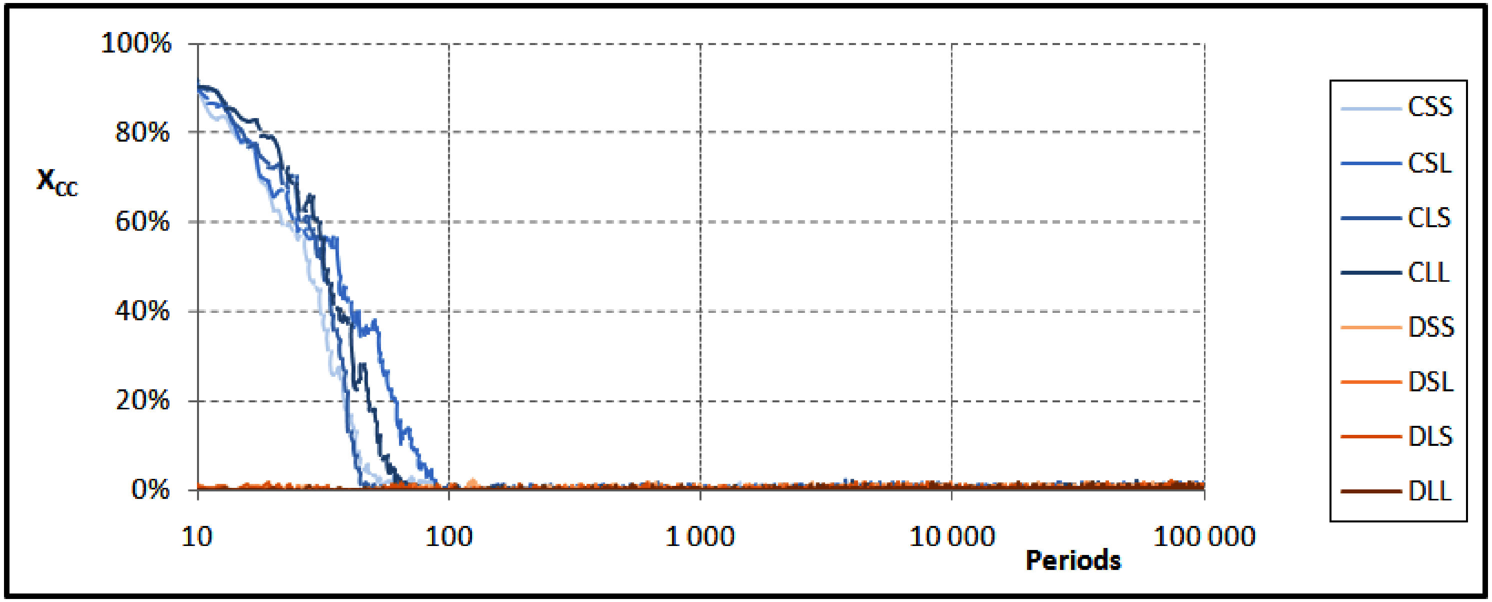
Expected lifespan of the individuals:

Short life	***expLife* = 5**
Long life	***expLife* = 50**
Intermediate life	***expLife* = 20**

Benchmark: $N = 400$, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

Short life: non-cooperative regime

x_{CC} : share of CC outcomes in a period

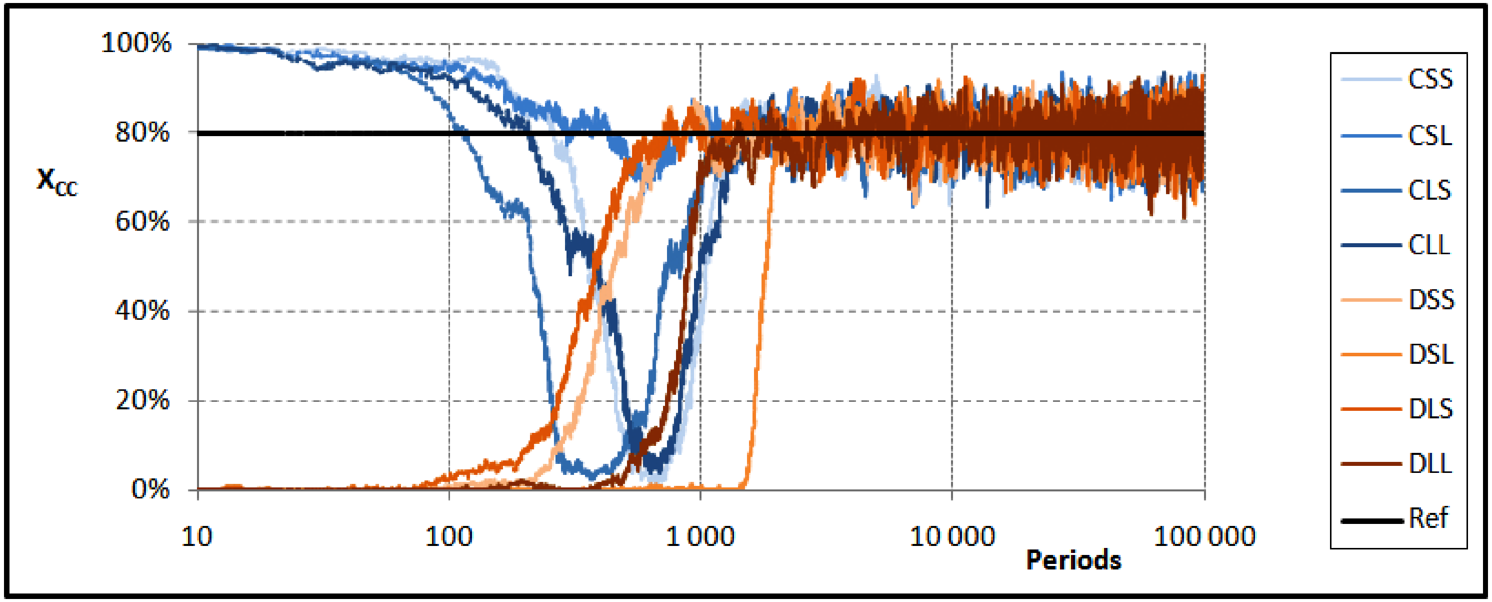


Evolution of the percentage of **CC outcomes** for 8 different runs, each run starting with the whole population using one of the 8 different strategies.

Parameterisation: $N = 400$, ***expLife* = 5**, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

Long life: cooperative regime

x_{CC} : share of CC outcomes in a period

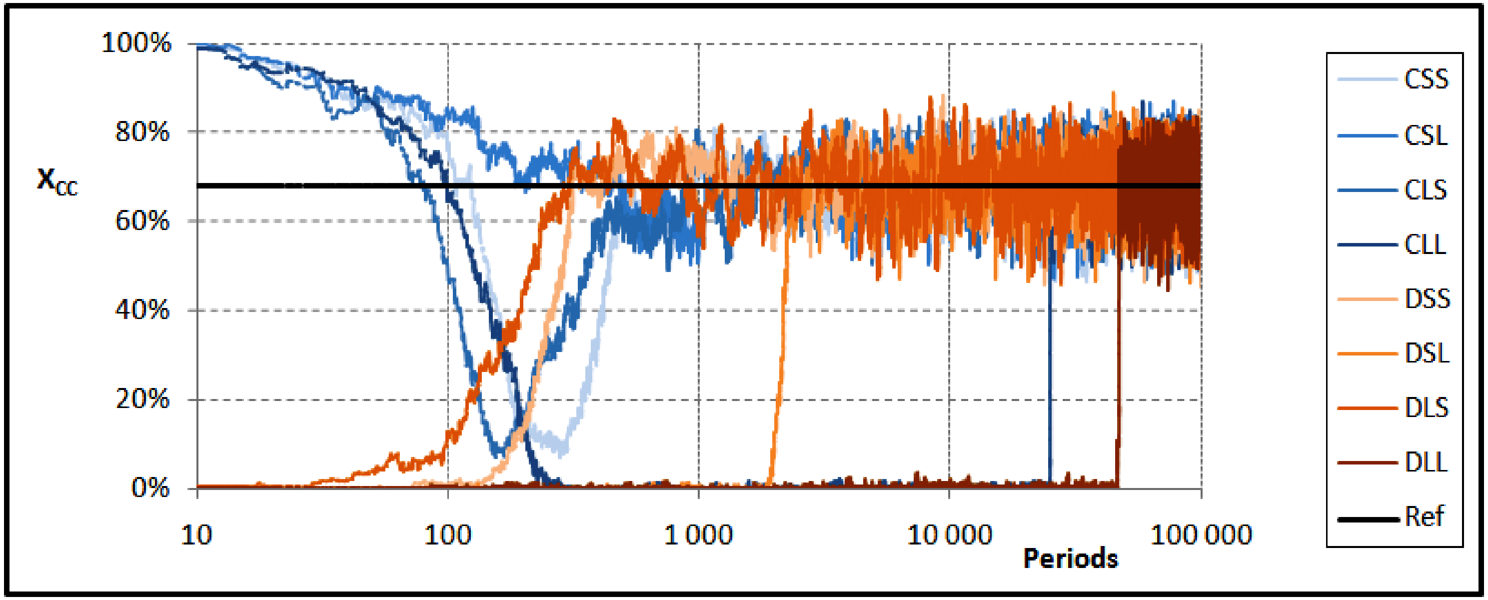


Evolution of the percentage of **CC outcomes** for 8 different runs, each run starting with the whole population using one of the 8 different strategies.

Parameterisation: N = 400, ***expLife* = 50**, μ = 0.05, T = 4, R = 3, P = 1, S = 0.

Intermediate life: dd-reg <-> cc-reg

x_{CC} : share of CC outcomes in a period



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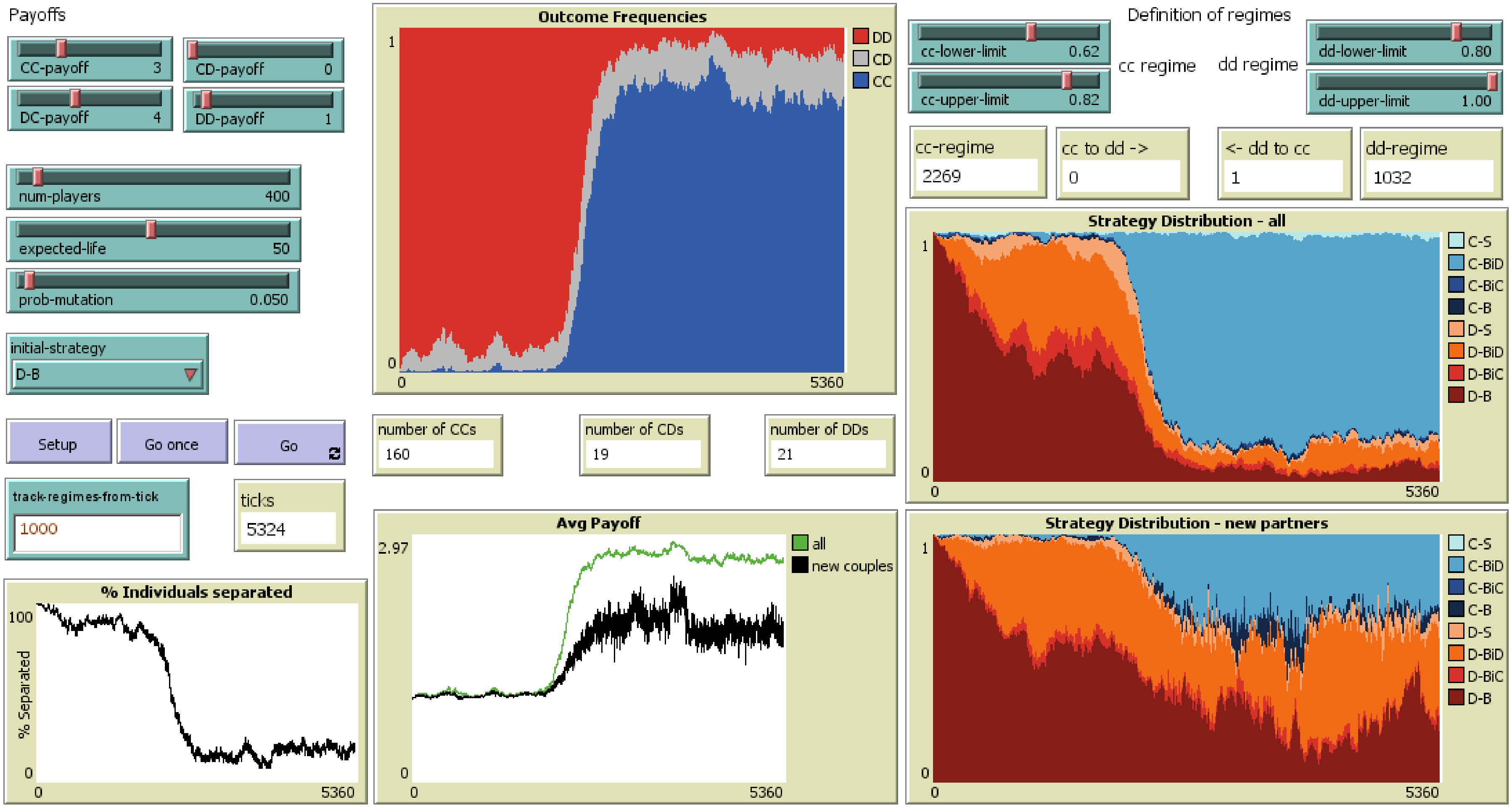
The 2 regimes



Payoffs

CC-payoff 3

CD-payoff 0

DC-payoff 4

DD-payoff 1

num-players 400

expected-life 50

prob-mutation 0.050

initial-strategy D-B

track-regimes-from-tick 1000

ticks 5324

Definition of regimes

cc-lower-limit 0.62

cc-upper-limit 0.82

dd-lower-limit 0.80

dd-upper-limit 1.00

cc regime

dd regime

cc-regime 2269

cc to dd -> 0

<- dd to cc 1

dd-regime 1032

number of CCs 160

number of CDs 19

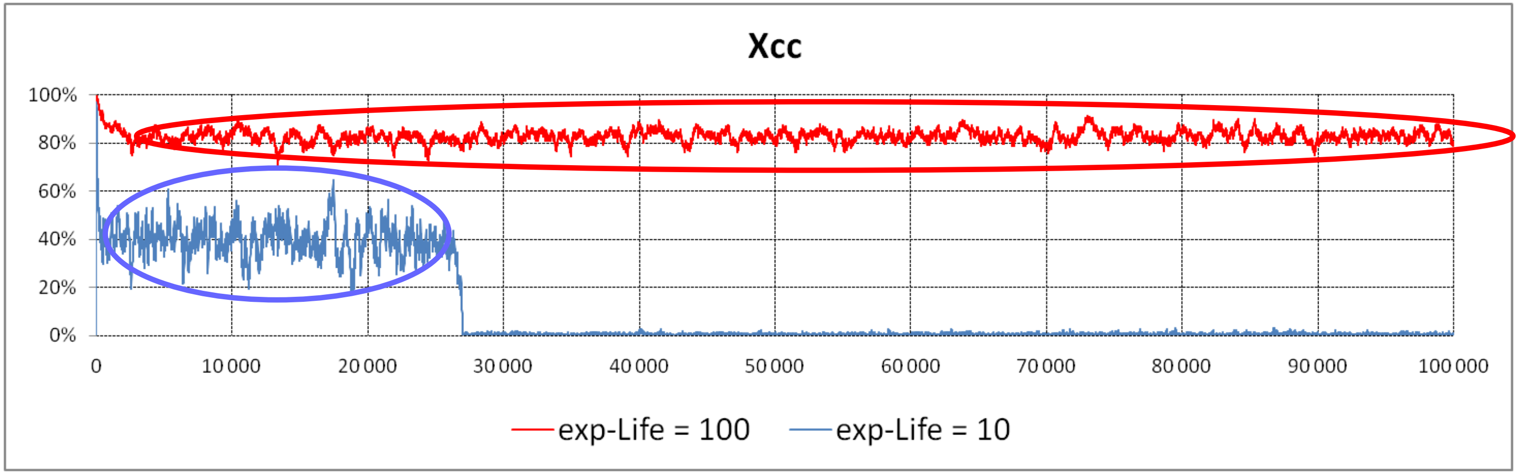
number of DDs 21

Characterising the 2 regimes

- Non-cooperative regime... Easy:

$$x_{DD} \geq 0.8$$

- (partially) Cooperative regime... not trivial



<u>Need to define a reference value</u>: $x_{CC}^{CR}(expLife)$

x_{CC} : share of CC outcomes in a period

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Mean-dynamics approximation

Time-homogeneous Markov chain

Ergodic if *expLife* $< \infty$ and $\mu > 0$.
Too many states, even for moderate values of *N*. (e.g. N=400 -> #S = 6.53734×10^{41})

Mean-dynamics approximation

When the population is very large, the law of large numbers enables us to:

- identify *ex ante* probabilities with *ex post* frequencies at the pairing stage,
- identify *expected* deaths with *average* deaths for each group of strategists,
- identify *ex ante* probabilities with *ex post* frequencies at the mutation stage.

This leads us to a deterministic set of 35 differential equations whose limiting behaviour may well depend on initial conditions.

Mean-dynamics approximation

$$\frac{dp_{rq}(t)}{dt} = \phi_{rq}\,\delta^2 p_{rq}(t) + y_r(t)\frac{y_q(t)}{y(t)} - p_{rq}(t)$$

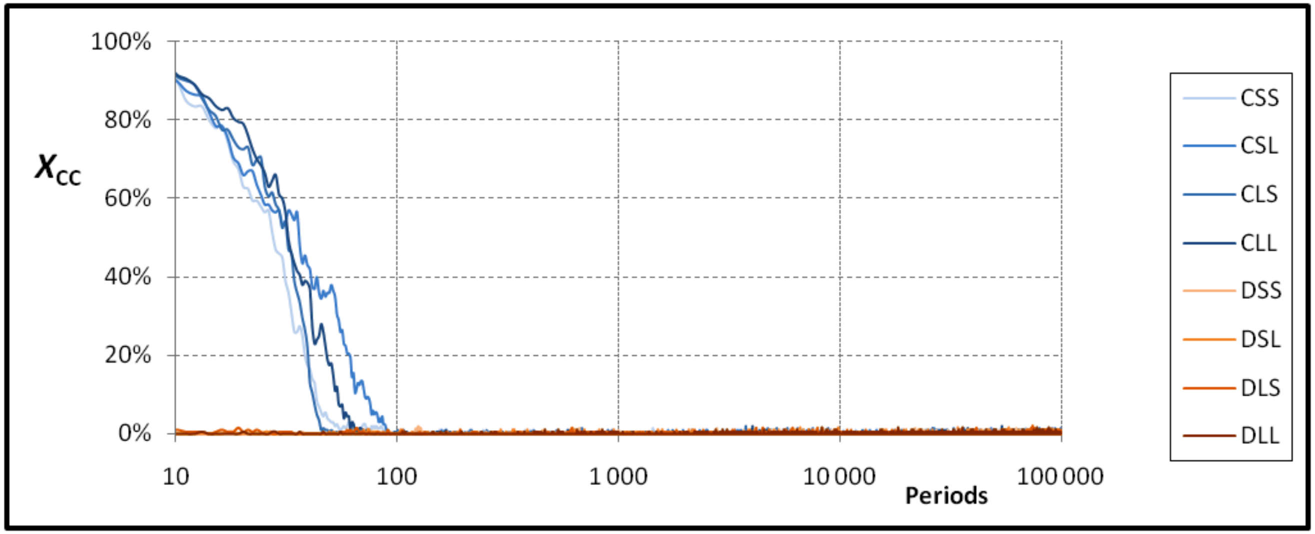
$$y_r = (1-\delta)\left[(1-\mu)\frac{p_r\overline{\pi_r}}{\bar{\pi}} + \mu m_r\right] + \delta^2 \sum_{i\in\Lambda}(1-\phi_{ri})p_{ri} + \delta(1-\delta)p_r$$

$$p_r = \sum_{i\in\boldsymbol{\Lambda}} p_{ri} \qquad\qquad \bar{\pi}(\boldsymbol{p}) = \sum_{i\in\Lambda} p_i \cdot \overline{\pi_i}(\boldsymbol{p})$$

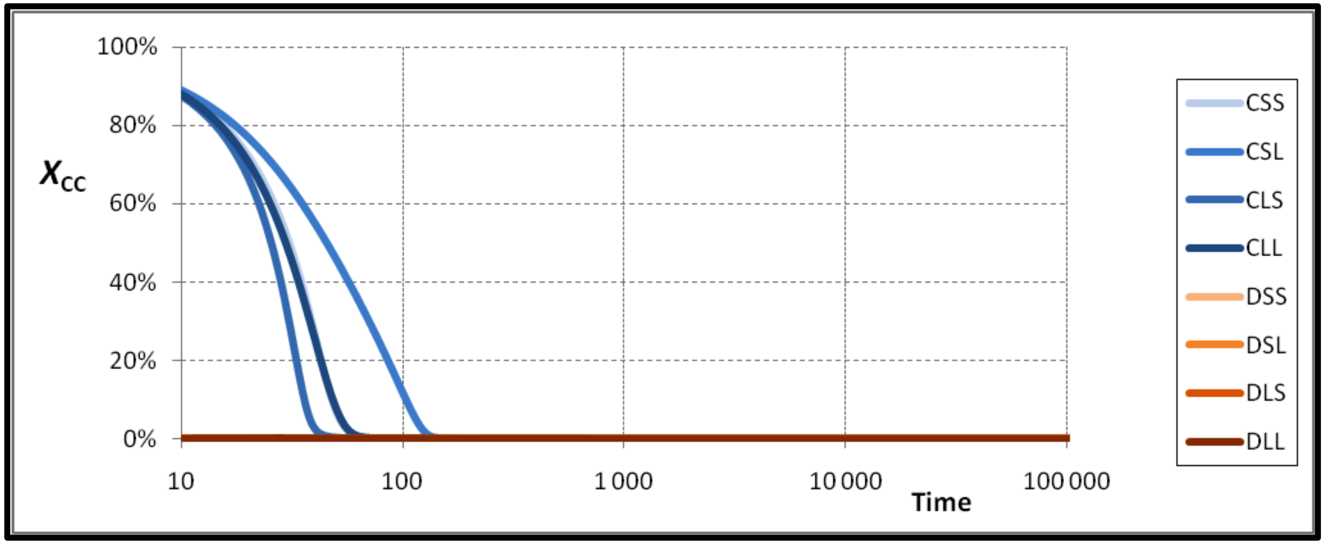
$$\overline{\pi_r}(\boldsymbol{p}) = \begin{cases} \dfrac{1}{p_r}\left(R\sum_{i\in\boldsymbol{\Lambda}_{\mathbf{C}}} p_{ri} + S\sum_{i\in\boldsymbol{\Lambda}_{\mathbf{D}}} p_{ri}\right) & \text{if } r \in \boldsymbol{\Lambda}_{\mathbf{C}} \\ \dfrac{1}{p_r}\left(T\sum_{i\in\boldsymbol{\Lambda}_{\mathbf{C}}} p_{ri} + P\sum_{i\in\boldsymbol{\Lambda}_{\mathbf{D}}} p_{ri}\right) & \text{if } r \in \boldsymbol{\Lambda}_{\mathbf{D}} \end{cases}$$

Evolution of the percentage of **CC outcomes** for 8 different runs, each run starting with the whole population using one of the 8 different strategies. Parameterisation: $N = 400$, **$expLife = 5$**, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

Stochastic simulations

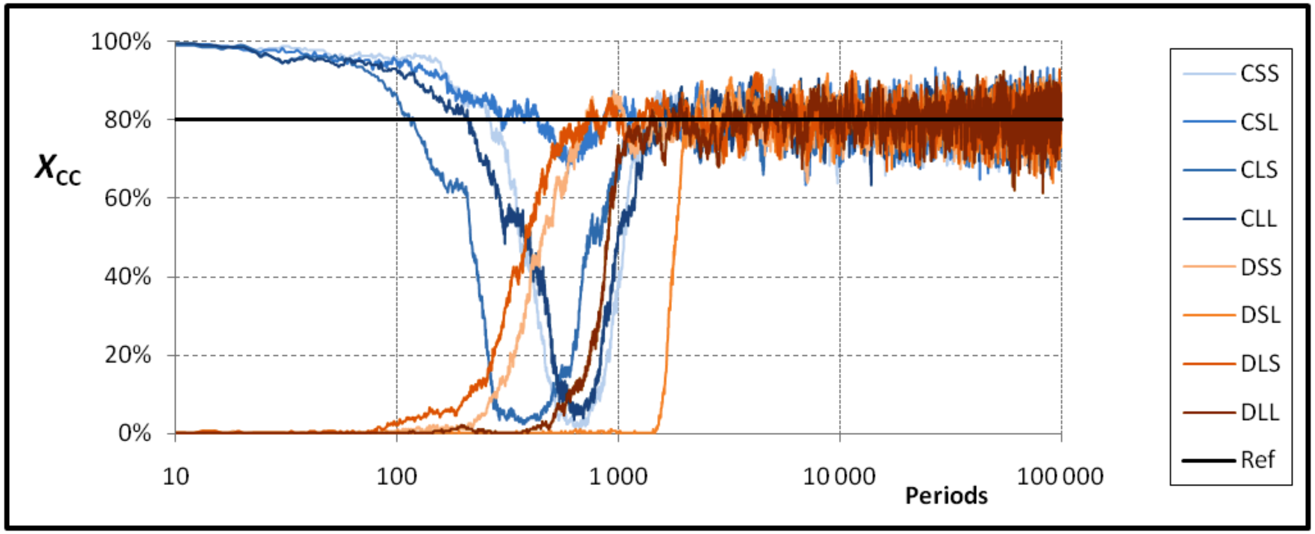


Mean-dynamics approximation



Evolution of the percentage of **CC outcomes** for 8 different runs, each run starting with the whole population using one of the 8 different strategies. Parameterisation: N = 400, ***expLife* = 50**, μ = 0.05, T = 4, R = 3, P = 1, S = 0.

Stochastic simulations



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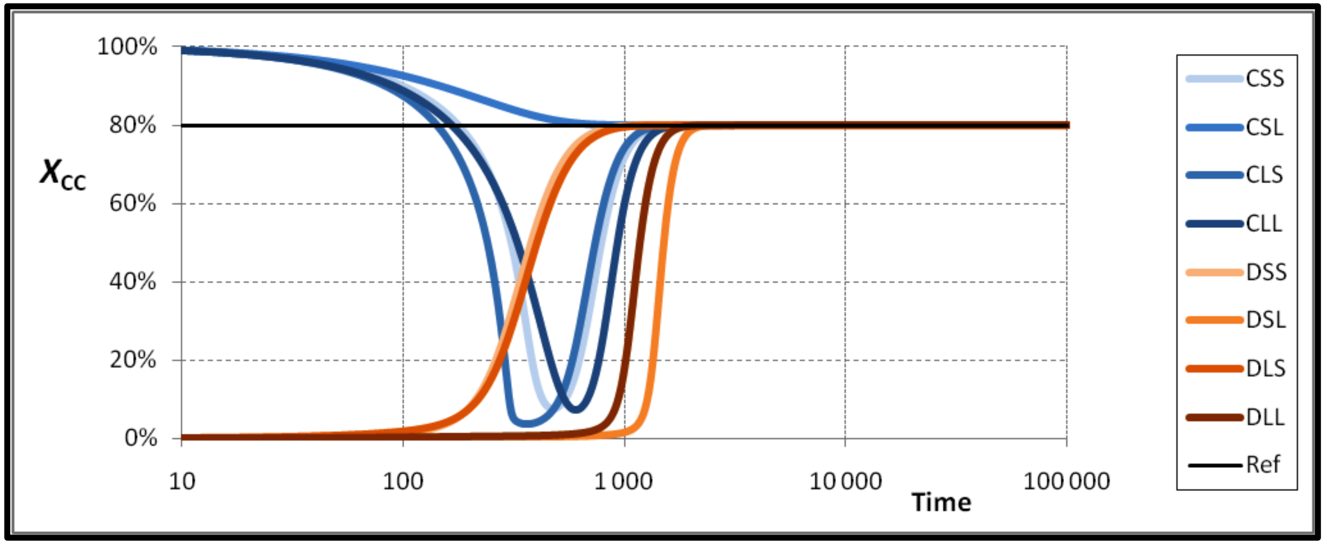
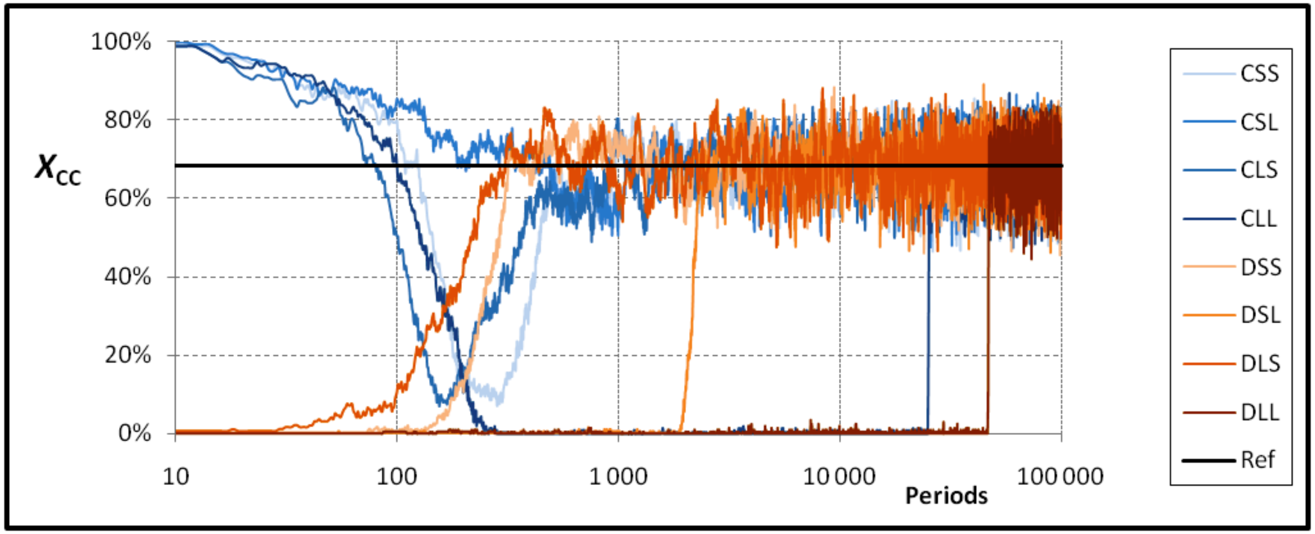
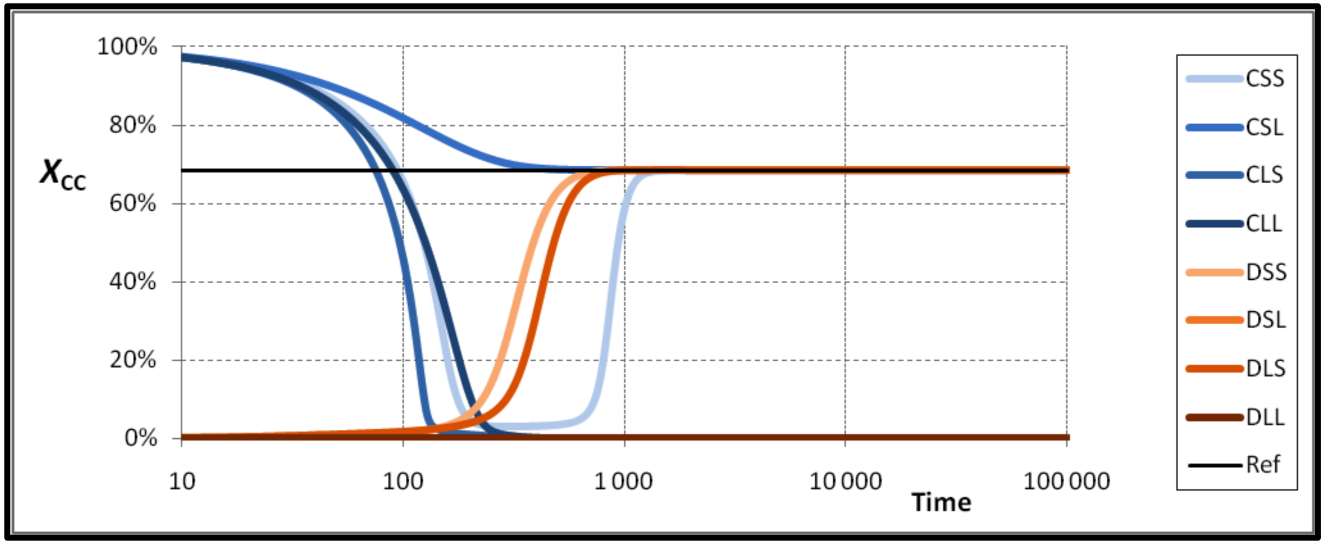


Evolution of the percentage of **CC outcomes** for 8 different runs, each run starting with the whole population using one of the 8 different strategies. Parameterisation: N = 400, **expLife = 20**, μ = 0.05, T = 4, R = 3, P = 1, S = 0.

Stochastic simulations



Mean-dynamics approximation

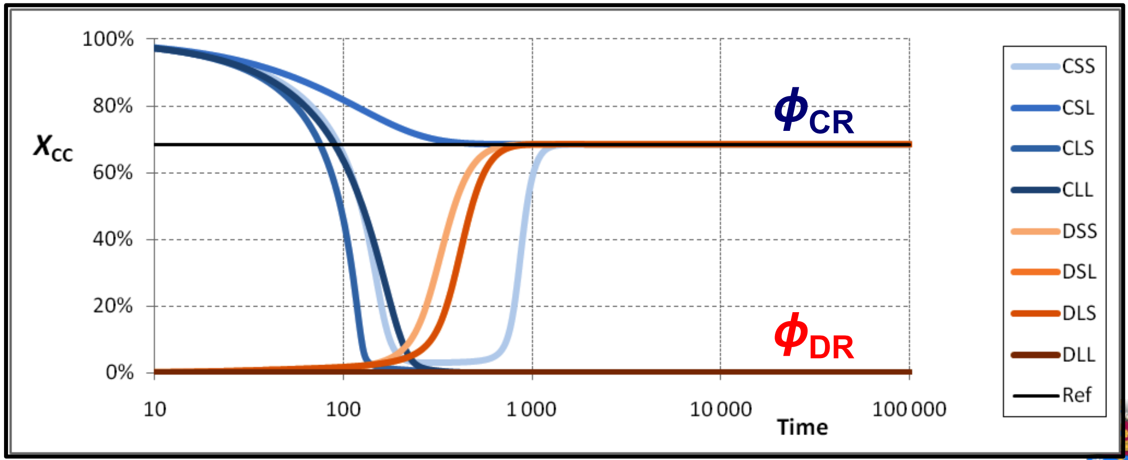


Mean-Dynamics approximation

Numerical exploration of the mean-dynamics equations starting from different initial conditions, and for different values of ***expLife*** and small $\mu > 0$ reveals:

- For expected life ***expLife*** < f_1, the MD equations converge to ϕ_{DR}.
- For expected life $f_1 \leq$ ***expLife*** < f_2 the MD equations converge either to ϕ_{DR}, or to ϕ_{CR}, characterised by high values of CC outcomes.
- For expected life $f_2 \leq$ ***expLife*** the MD equations converge to ϕ_{CR}.

Example of initial conditions for which the MD equations converge to
a) the cooperative attractor ϕ_{CR}.
b) the defective attractor ϕ_{DR}.



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- For expected life $f_2 \leq$ ***expLife*** the MD equations converge to ϕ_{CR}.

We refer to the fraction of CC outcomes in ϕ_{CR} as x_{CC}^{CR}. The longer the expected life, the greater the basin of attraction of ϕ_{CR} and the higher the value of x_{CC}^{CR}. We use the value x_{CC}^{CR} to define the (partially) cooperative regime as:

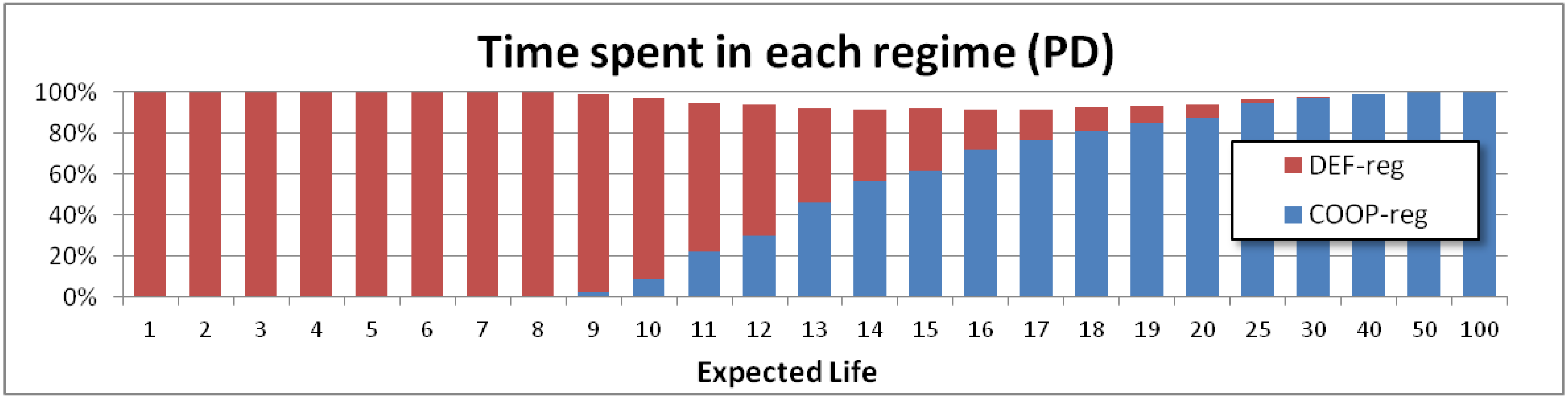
$$\left| x_{CC} - x_{CC}^{CR} \right| \leq 0.1$$

Mean-Dynamics approximation

expLife	x_{CC}^{CR}	**Cooperative Regime**	**Non-Cooperative Regime**
8	-	-	$x_{DD} \geq 0.8$
9	33.16%	$0.2616 \leq x_{CC} \leq 0.4616$	
10	43.90%	$0.3390 \leq x_{CC} \leq 0.5390$	
15	61.57%	$0.5157 \leq x_{CC} \leq 0.6157$	
20	68.43%	$0.5843 \leq x_{CC} \leq 0.7843$	
25	72.30%	$0.6230 \leq x_{CC} \leq 0.8230$	
30	74.85%	$0.6485 \leq x_{CC} \leq 0.8485$	
40	78.04%	$0.6804 \leq x_{CC} \leq 0.8804$	
50	80.00%	$0.7000 \leq x_{CC} \leq 0.9000$	
100	84.24%	$0.7424 \leq x_{CC} \leq 0.9424$	

Parameterisation: $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

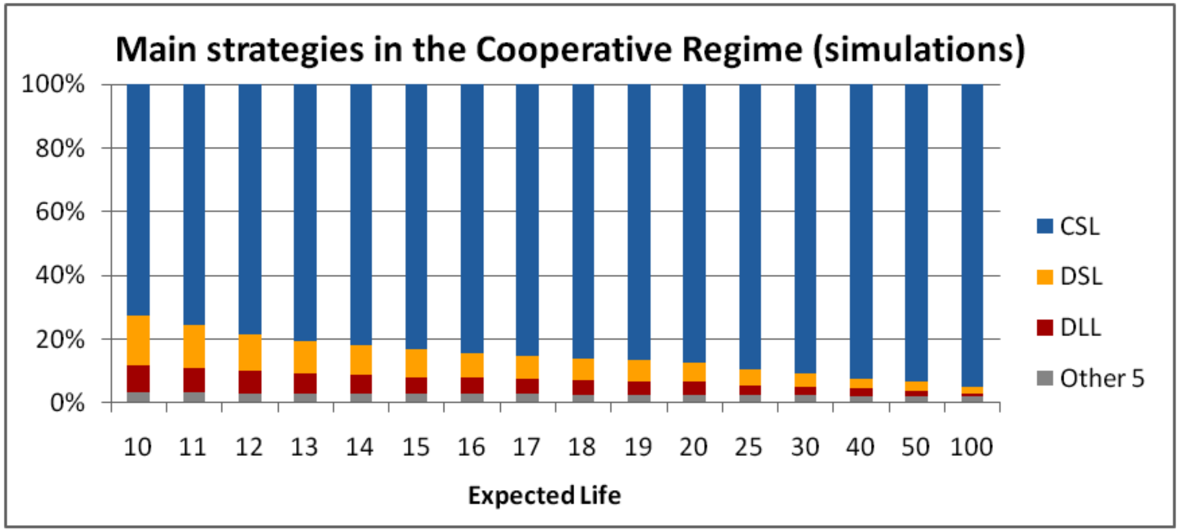
Results

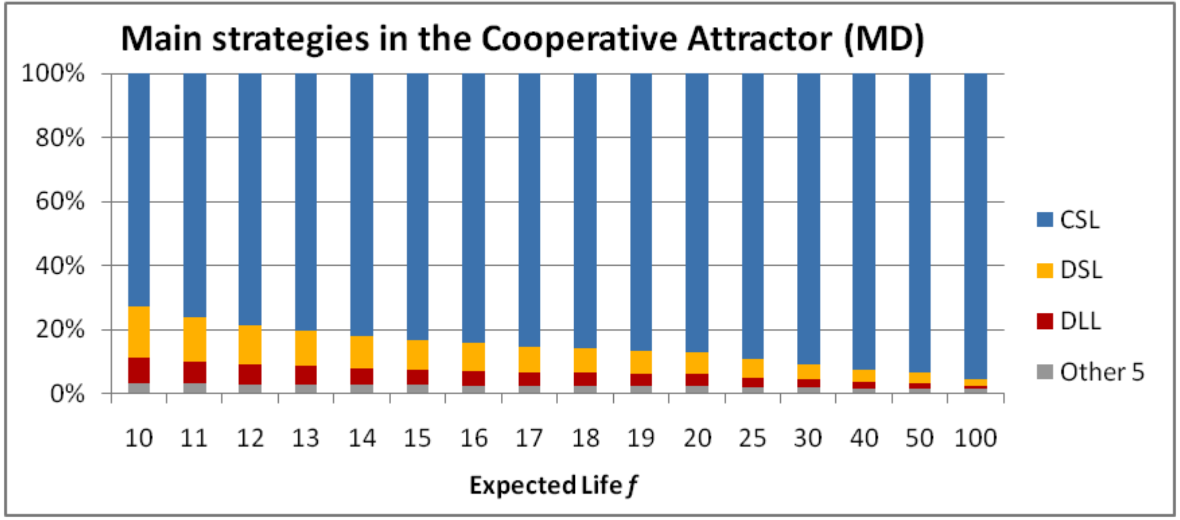


Fraction of periods spent in **the non-cooperative** and in **the cooperative** regimes as a function of the expected life ***expLife***.

The values in each column are compiled over 10^3 simulation runs.
Every run measured between periods $3 \cdot 10^3$ and 10^4, with random initial conditions.
Parameterisation: $N = 400$, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

Distribution of Strategies in the cooperative regime



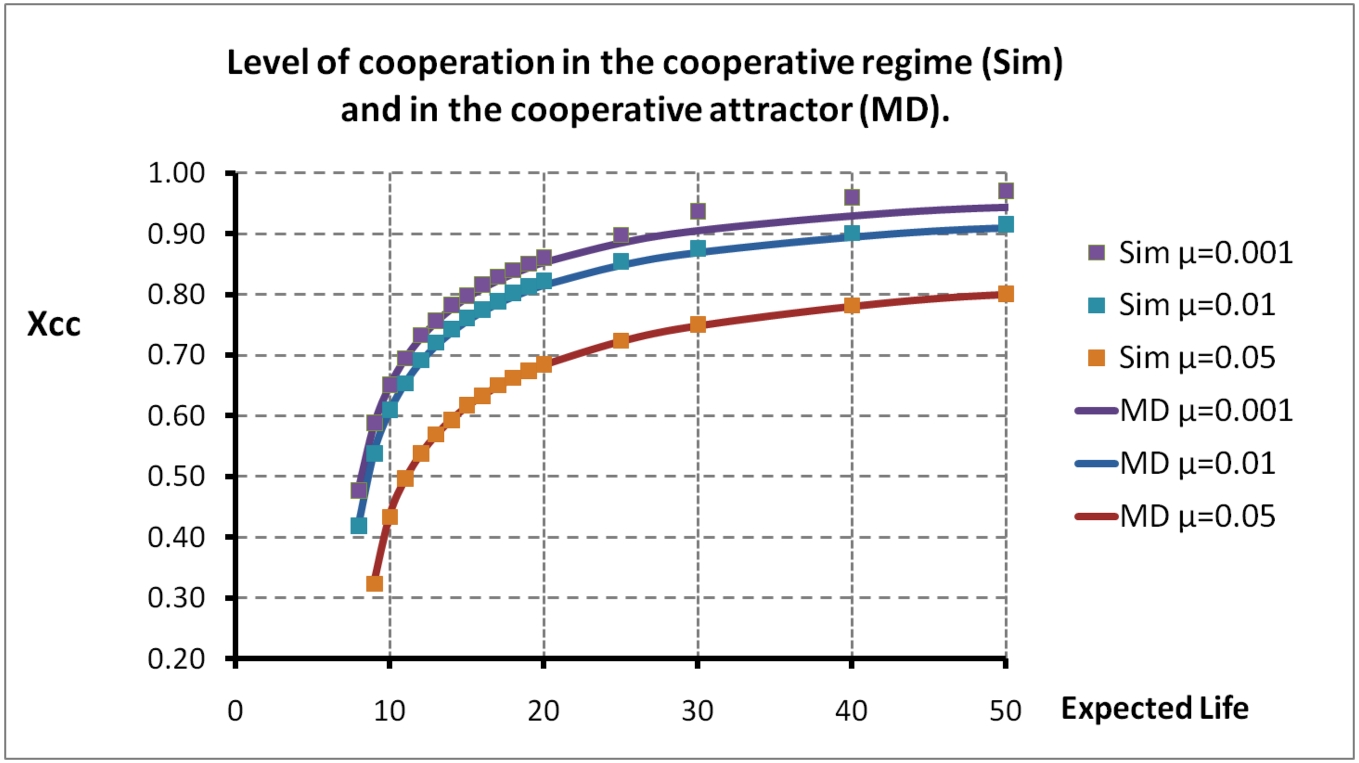


The values in each column are compiled over 1000 simulation runs.

Every run measured between periods $3 \cdot 10^3$ and 10^4, with random initial conditions.

Parameterisation:
$N = 400$, $\mu = 0.01$,
$T = 4$, $R = 3$, $P = 1$, $S = 0$.

Level of cooperation



Average values of the level of cooperation (x_{CC})
in the cooperative regime (stochastic simulations)
and in the cooperative attractor (Mean Dynamics)
as a function of the individuals' expected life *expLife*, for different values of μ.

Parameterisation: $N = 400$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

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- Introduction
- The question and the approach
- The model
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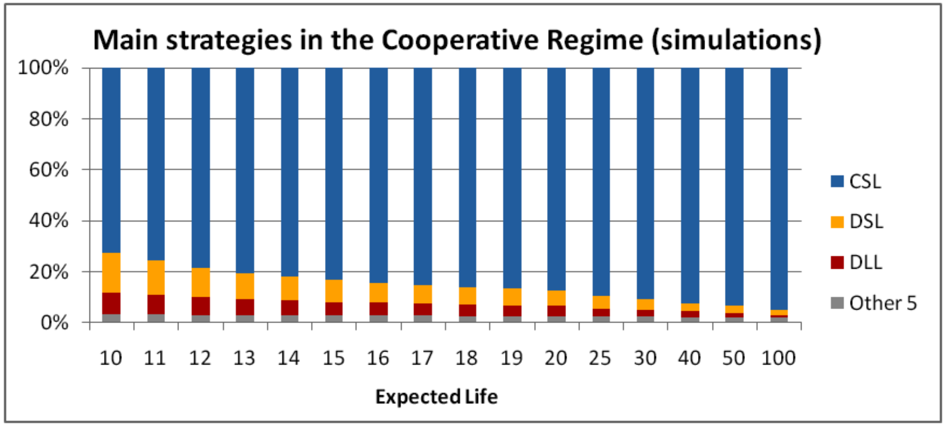
Reduced mean-dynamics approximation

Reduced mean-dynamics approximation to try to derive a closed-form solution for the level of cooperation in the cooperative regime for arbitrary payoffs and arbitrary mutation.

$$x_{CC}^{CR}(expLife, T, R, P, S)$$

Focus only on the strategies that the numerical simulations single out as the most prevalent in the cooperative regime:

CSL , **DSL** and **DLL**.



Also, these strategies are the only ones that can be part of a (partially) cooperative Nash equilibrium (appropriately defined for this dynamic context).

Reduced mean-dynamics approximation

$$\dot{x}_{CC} = \frac{y_C^2}{y} + (1-\beta)^2 x_{CC} - x_{CC}$$

$$y = 1 - (1-\beta)^2 x_{CC}$$

$$\dot{x}_{DD} = \frac{y_D^2}{y} - x_{DD}$$

$$y_C = \beta\left[(1-\mu)\frac{\pi_C}{\pi_C + \pi_D} + \frac{\mu}{4}\right] + (1-\beta)(\beta x_{CC} + \frac{x_{CD}}{2})$$

$$\dot{x}_{CD} = 2\frac{y_C\, y_D}{y} - x_{CD}$$

$$y_D = \beta\left[(1-\mu)\frac{\pi_D}{\pi_C + \pi_D} + \frac{3\mu}{4}\right] + (1-\beta)(x_{DD} + \frac{x_{CD}}{2})$$

Only 3 state variables (type of interaction) that add up to one.

$$\pi_C = R\, x_{CC} + S\frac{x_{CD}}{2}$$

$$\pi_D = P\, x_{DD} + T\frac{x_{CD}}{2}$$

Reduced mean-dynamics approximation

x_{CD} : share of CD, DC outcomes in a period

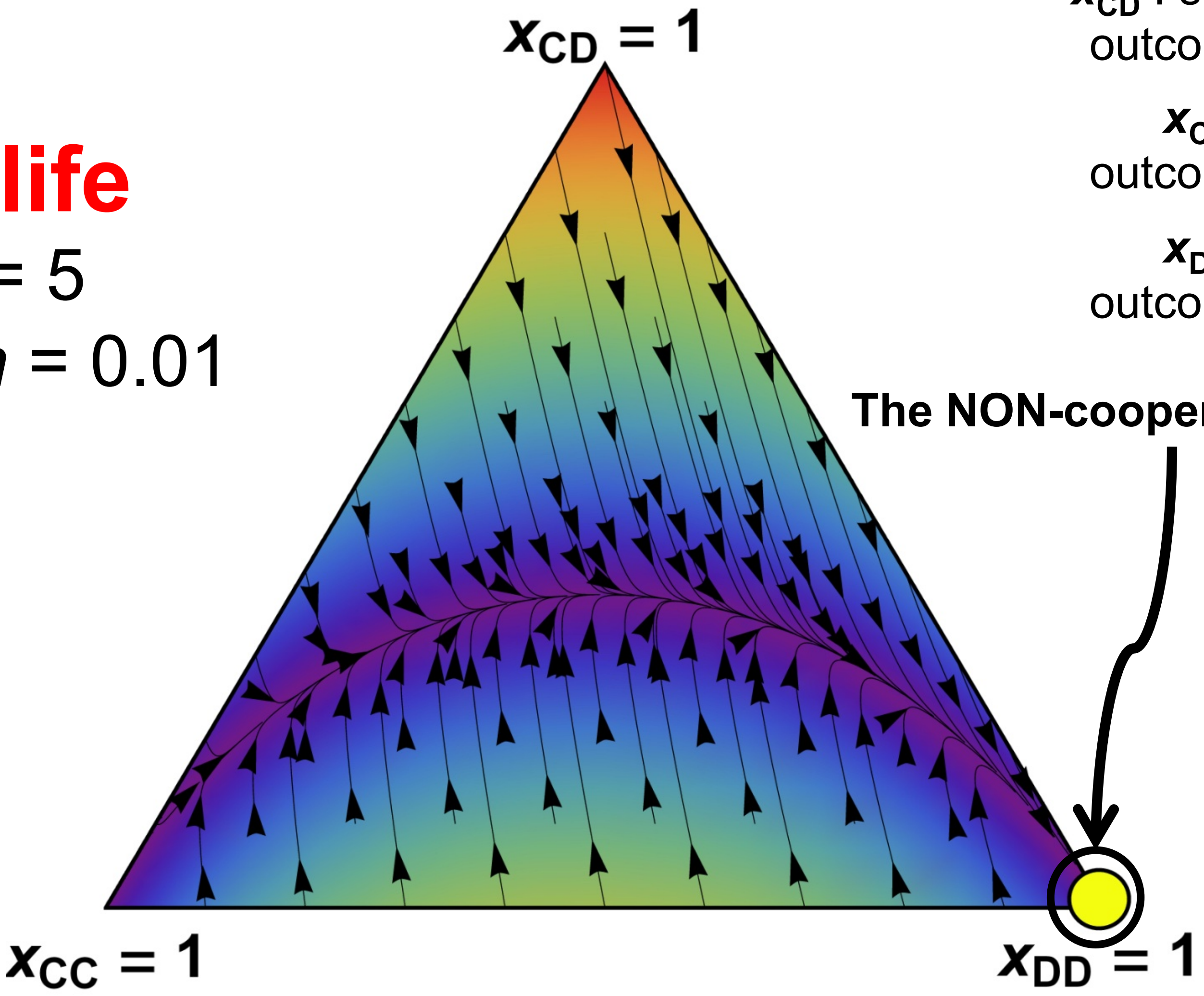
x_{CC} : share of CC outcomes in a period

x_{DD} : share of DD outcomes in a period

Short life

expLife = 5

Mutation = 0.01



The NON-cooperative regime

Reduced mean-dynamics approximation

x_{CD} : share of CD, DC outcomes in a period

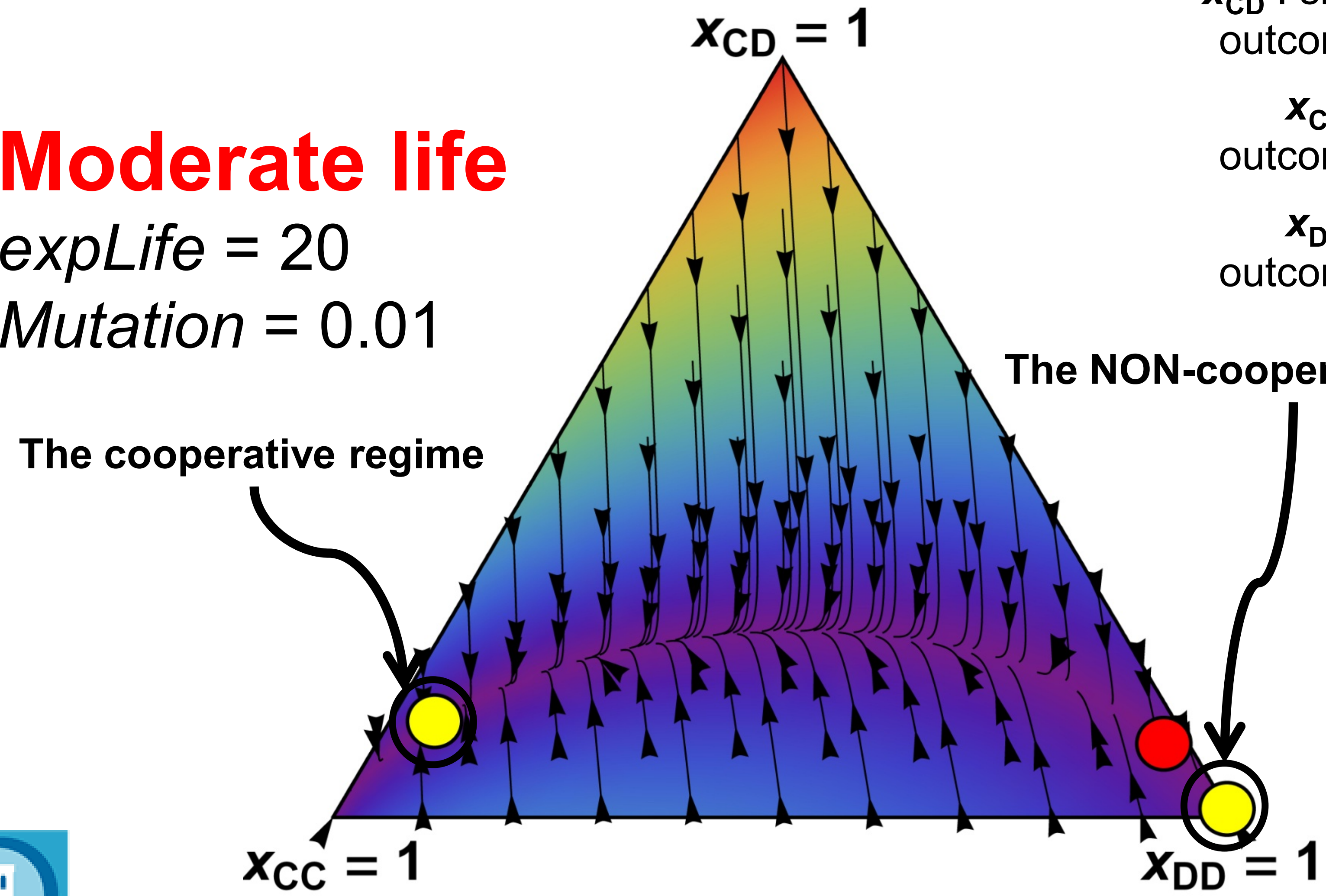
x_{CC} : share of CC outcomes in a period

x_{DD} : share of DD outcomes in a period

Moderate life

expLife = 20

Mutation = 0.01



Reduced mean-dynamics approximation

x_{CD} : share of CD, DC outcomes in a period

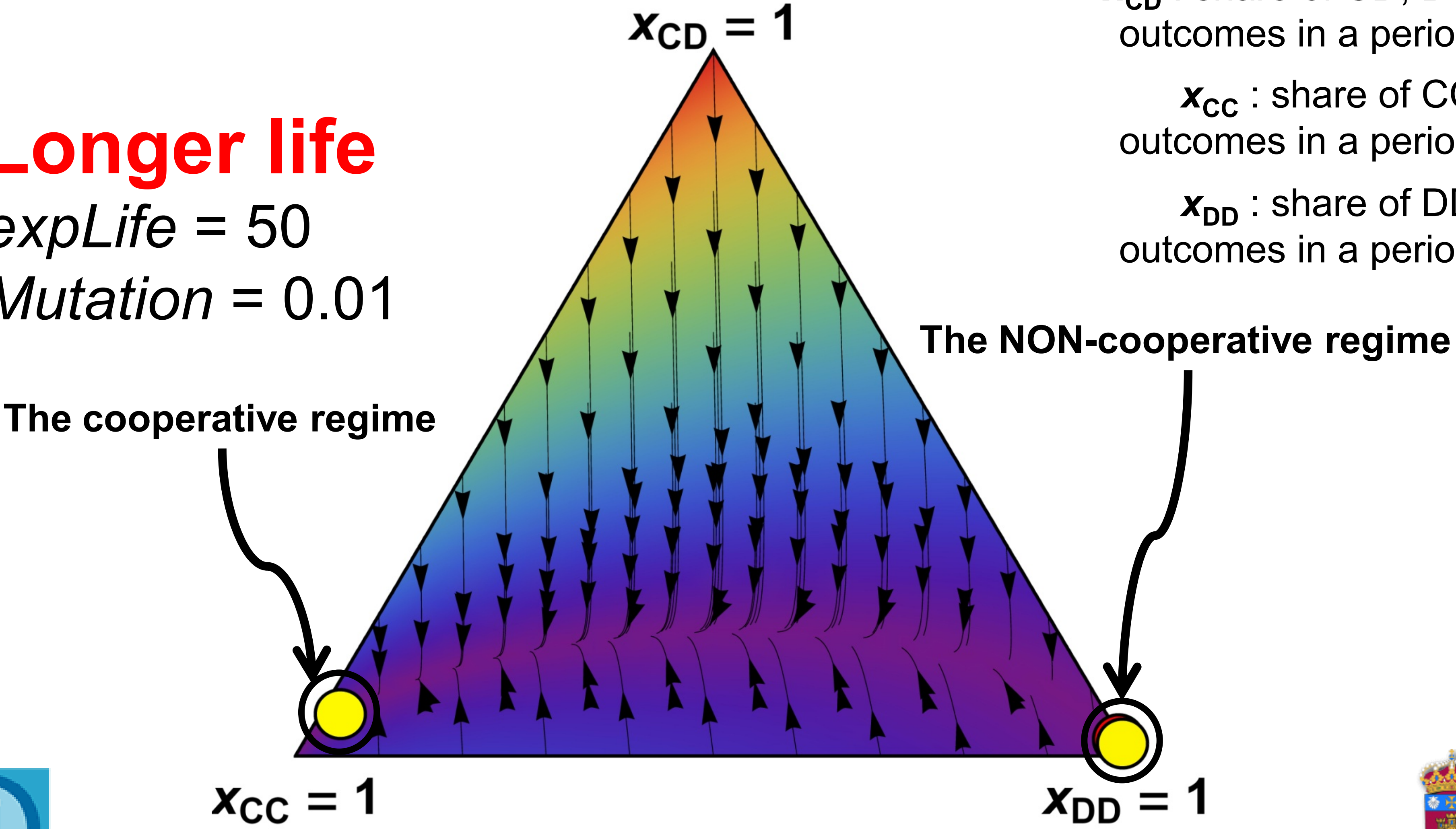
x_{CC} : share of CC outcomes in a period

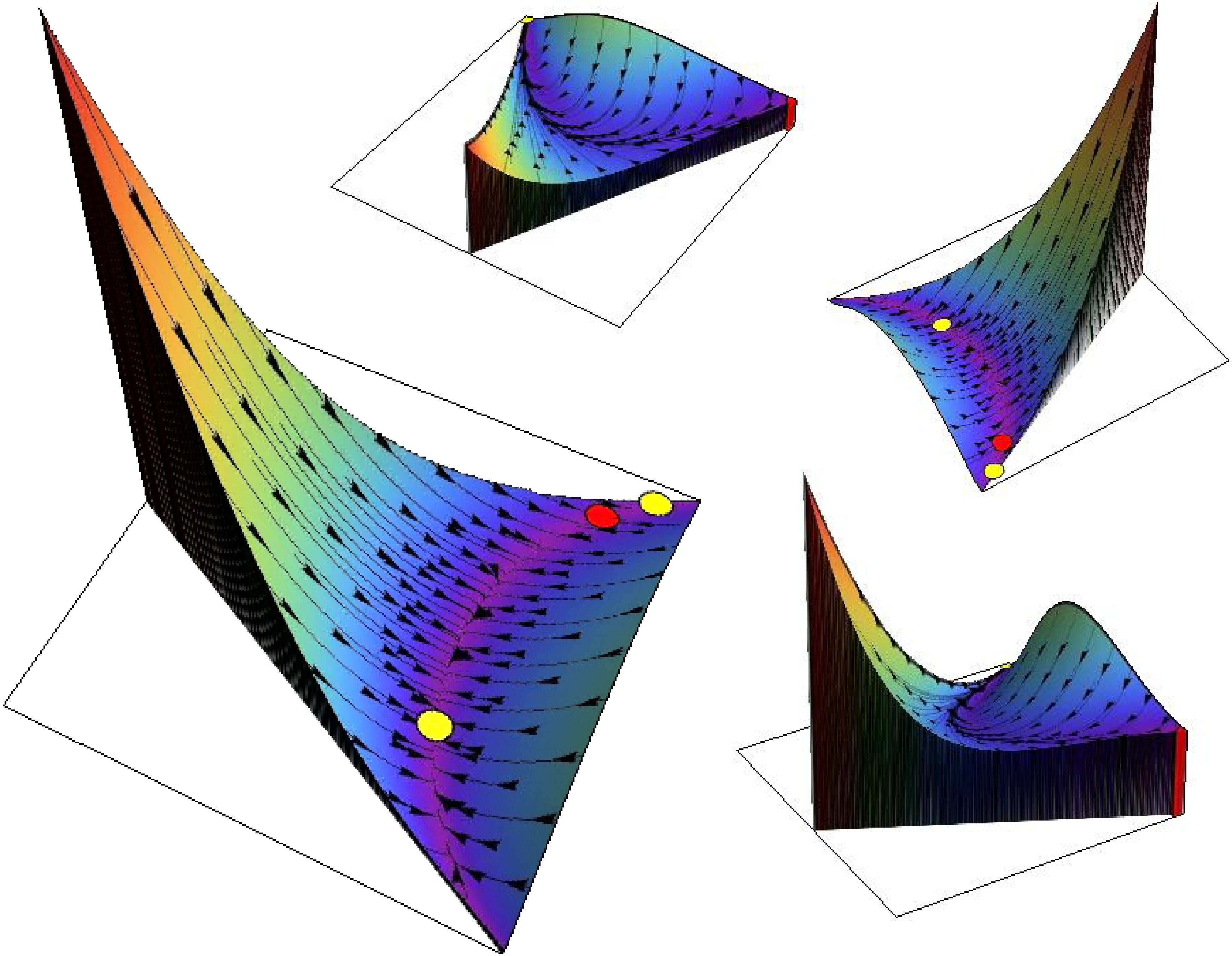
x_{DD} : share of DD outcomes in a period

Longer life

expLife = 50

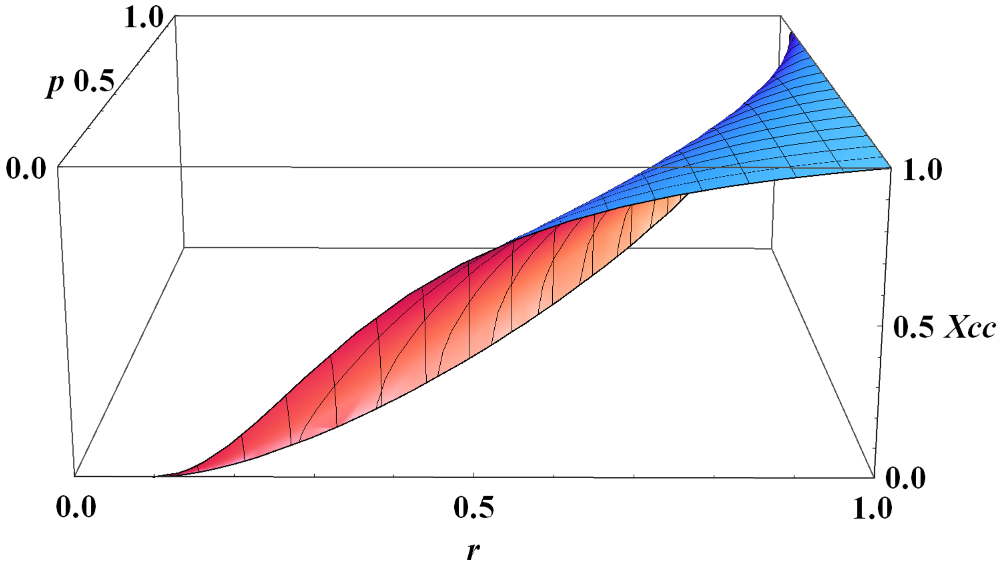
Mutation = 0.01





Reduced mean-dynamics approximation

$$x^*_{CC} = \frac{m^2 + \delta(1-2p)\big(\delta - 2(1-r)\big) + |m - \delta(1-2p)|\sqrt{(m-\delta)^2 - 4rp\delta}}{2\delta\left(m^2(1-\delta) + \delta(r-p)^2\right)}$$



where $\delta = \left(1 - {}^{1}/_{expLife}\right)^2$

$m \equiv 1 - r - p$

$0 < p = \dfrac{P - S}{T - S} < r = \dfrac{R - S}{T - S} < 1$

(No mutation)

Reduced mean-dynamics approximation

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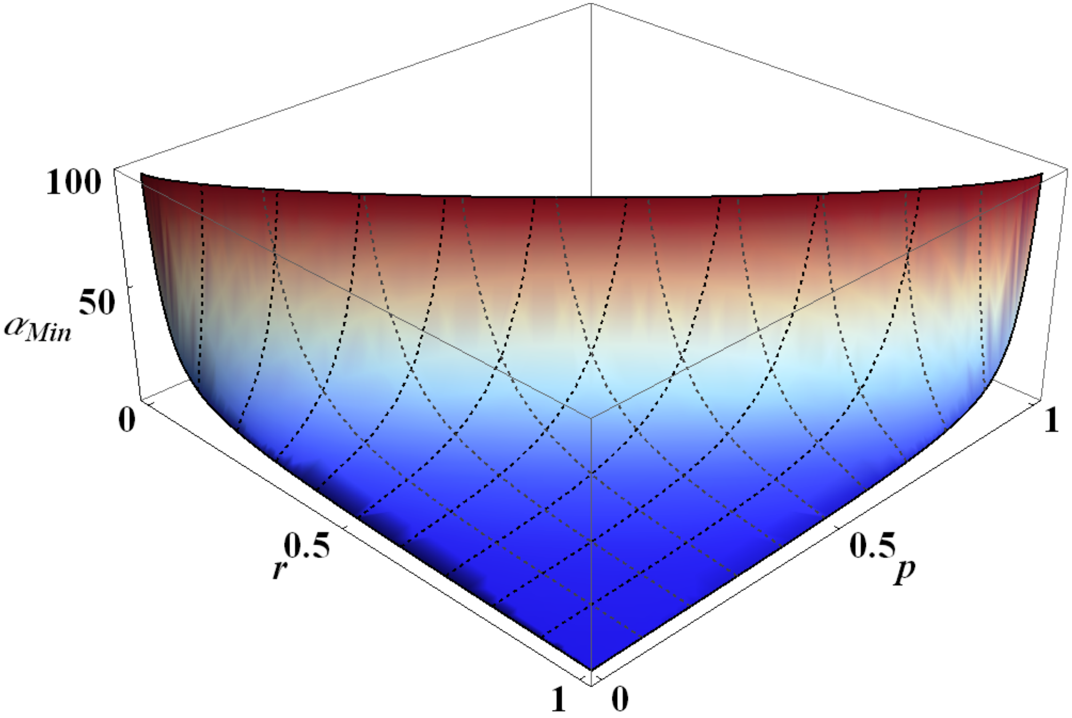
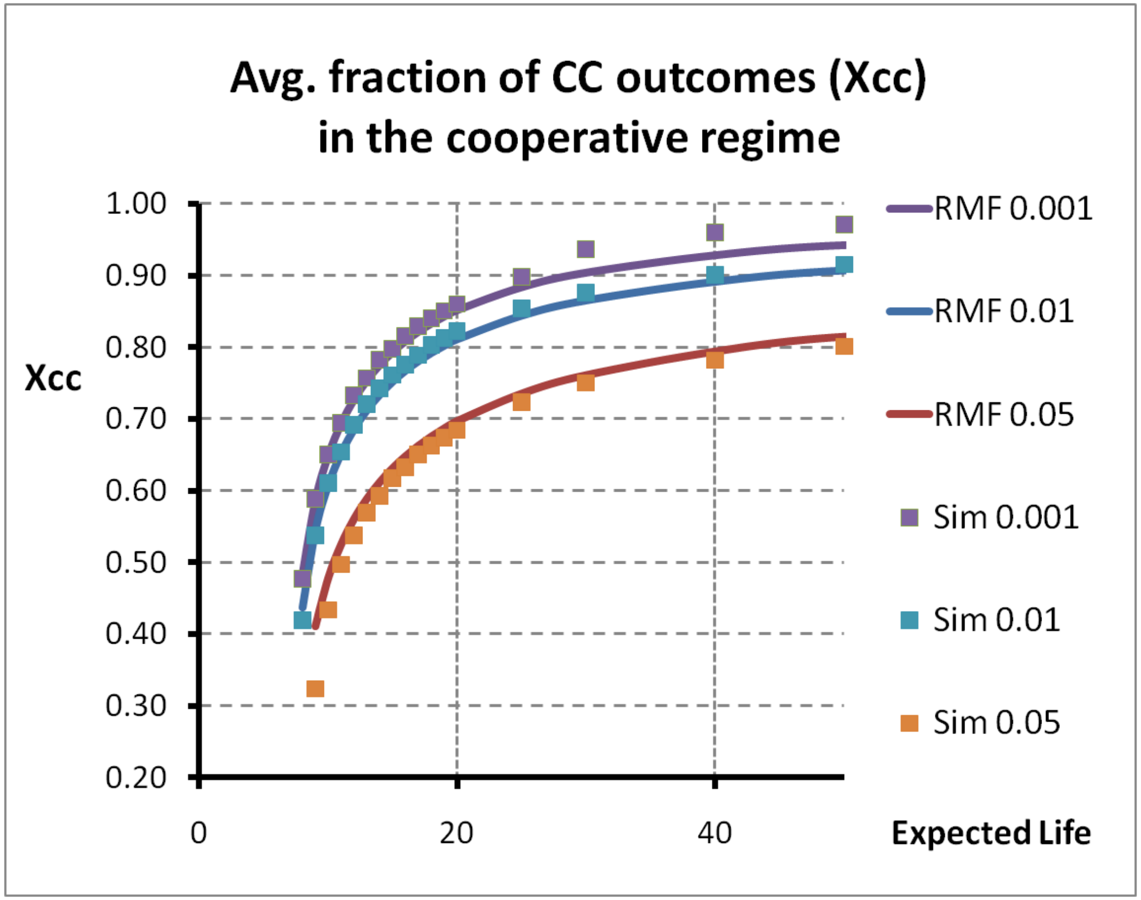
$$expLife_{Min} = \frac{1}{1 - \sqrt{rp} - \sqrt{(1-r)(1-p)}}$$



where $$0 < p = \frac{P-S}{T-S} < r = \frac{R-S}{T-S} < 1$$

(No mutation)

Reduced mean-dynamics approximation



Average values of the level of cooperation (x_{CC}) in the cooperative regime as a function of the individuals' expected life *expLife*, both in the reduced mean-dynamics approximation and in the stochastic simulations with different values of μ.

Parameterisation: $N = 400$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

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Conclusions

- A simple mechanism of conditional dissociation can explain the evolutionary emergence of cooperation.
- Key factor: expected lifespan of the individuals.
  It is sufficient that lifespans are only moderately long.
- Cooperative regime mostly composed of "*conditional dissociators*", *i.e.* cooperators that respond to defection by leaving.
- We have been able to derive a closed-form solution for arbitrary payoffs.

Leaving undesirable partners

- A sufficient condition to explain the evolutionary emergence of cooperation

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Paper: http://dx.doi.org/10.1016/j.jedc.2014.06.007
Online model: http://luis.izqui.org/models/LeLeLe/index.html
5-minute video: http://luis.izqui.org/papers/Izquierdo_Izquierdo_Vega-Redondo_2014.html